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Probabilistic representation in syllogistic reasoning: A theory to integrate mental models and heuristics

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ABSTRACT

This paper presents a new theory of syllogistic reasoning. The proposed model assumes there are probabilistic representations of given signature situations. Instead of conducting an exhaustive search, the model constructs an individual-based "logical" mental representation that expresses the most probable state of affairs, and derives a necessary conclusion that is not inconsistent with the model using heuristics based on informativeness. The model is a unification of previous influential models. Its descriptive validity has been evaluated against existing empirical data and two new experiments, and by qualitative analyses based on previous empirical findings, all of which supported the theory. The model's behavior is also consistent with findings in other areas, including working memory capacity. The results indicate that people assume the probabilities of all target events mentioned in a syllogism to be almost equal, which suggests links between syllogistic reasoning and other areas of cognition.

All fools are poets; this the Prefect feels; and he is merely guilty of a non distributio medii in thence inferring that all poets are fools.

[— Edgar Allan Poe, *The Purloined Letter* (1845)]

1. Introduction

Reasoning is intended to derive reasonable conclusions from premises. Given the assertions that "The Kyotoite are Japanese" and "The Japanese are Asian," it is reasonable to conclude that "The Kyotoite are Asian." In this case, the relation is *transitive*: if $K \rightarrow J$ and $J \rightarrow A$, then $K \rightarrow A$. However, if one knows that "The Kyotoite are suave," it is illogical to infer that "Suave people are Kyotoite." A relation is *symmetric* if $X \rightarrow Y$ implies $Y \rightarrow X$, but such symmetrical derivations are not licensed in logic. As such, some inferences are logically valid, and others are invalid; some are easy, and others are difficult. The difficulty of inference depends, at least partly, on its logical form, but an error-prone argument can sometimes be obvious with a slight change in wording (e.g., using familiar terms). At the same time, difficulty of inference must relate to other types of thinking, because if nothing else, reasoning must be carried out in working memory. Any comprehensive psychological theory of reasoning must address these issues, that is, why some inferences are difficult and how this relates to other areas of cognition. The current paper proposes one such attempt, along with the novel idea of *probabilistic representation*. Before going into detail, however, I first motivate two issues regarding cognitive architecture and inferential structure: mental representations and symmetry, which will feature strongly in what follows.

The current theory (*probabilistic representation theory* hereafter) proposes dual mental representations: probabilistic representations and individual-based mental models.[1] This is based on the hypothesis that people have several thinking modes. We sometimes take a *summary view* with probabilistic representations when, for example, we are seeking some rules or tendencies that are useful for predictive inference. In this heuristic mode, we think and talk about probable relations between classes of events or objects (e.g., "I know the Kyotoite are suave [by and large]."). In contrast, when critically testing a hypothesis or thinking counterfactually, we take a *distinctive view* with individual-based representations. In this analytic mode, we talk about stricter (i.e., more logical) rules, sometimes focusing on exceptions (e.g., "One of my acquaintances is Kyotoite, but he is not suave; so, I don't think this is true."). In this way, we can easily switch views according to factors such as context, situation, motivation, and purpose. These two distinctive views, I

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[1] This general word does not necessarily indicate the same thing that is meant by the mental model theory introduced later.

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assume, depend on different representations: *continuous* (i.e., probabilistic) and *discrete* (i.e., individual-based).

Probabilistic representation theory supposes the summary view precedes the distinctive, because the summary view is based on heuristic processes but the distinctive view is based on deliberate processes. People first have a probabilistic intuition, and next construct mental models based on that intuition that serves for logical tests. As a result, the distinctive view is affected by the summary view, in that, people's probabilistic intuitions restrict how they test logical relations. In modeling this mechanism, a set of discrete mental models is a summary representation of a primitive continuous probabilistic model, and not the other way around. This implementation is unique among probabilistic approaches that have been proposed for deductive reasoning. One previous model (Chater & Oaksford, 1999) did not propose any internal representations, and the others (Guyote & Sternberg, 1981; Johnson-Laird, Legrenzi, Girotto, Legrenzi, & Caverni, 1999; Khemlani, Lotstein, Trafton, & Johnson-Laird, 2015) assume the priority of discrete models, introducing probabilistic behavior by allocating numerals (i.e., probability values) to discrete models.

This aspect of the theory is an extension of recent approaches to reasoning based on probability (e.g., Chater & Oaksford, 1999; Evans & Over, 2004; Oaksford & Chater, 2007) called the *new paradigm* in the psychology of reasoning (Elqayam & Over, 2013; Over, 2009). Although logic guides deductive reasoning, the idea that deduction depends on logic as a normative theory of human reasoning is now an "ancient proposal" (Johnson-Laird, Khemlani, & Goodwin, 2015, p. 201). After the 1990s, many researchers moved to probabilistic approaches to reasoning. In these approaches it is usually presupposed that degrees of certainty or belief correspond to subjective probabilities, and the validity of an argument is assessed via the probabilistic validity, or *p-validity*, proposed by Adams (1975): the uncertainty (i.e., the complement of the probability) of a *p*-valid conclusion does not exceed the sum of the uncertainties of the premises. This presupposition implicitly requires each proposition to retain its probability (at any time in any context, in principle) to enable probabilistic inference as follows:

The Kyotoite are suave.	(prob = 0.85)
The suave are ...	(prob = 0.43)
...	(prob = 0.05)
∴ The Kyotoite are ...	(prob = ...)

This actually places an excessive load on the working memory, especially when forming a chain of inferences, because an extra piece of information about probability must be retained for each statement. Moreover, even a couple of premises can result in innumerable (*p*-)valid (but vapid) conclusions (see, Johnson-Laird et al., 2015). Thus, a model based on a system of *p*-validity (as well as a standard binary logic) can generate serious concerns at the algorithmic level about the feasibility of a model implementing (deductive) reasoning. It seems reasonable to suppose that people discretize (i.e., simplify) their degrees of belief in each proposition at some point in time. For example, a statement with a probability of 95% or higher may be regarded as just a "true" statement somewhere in the course of the reasoning process. In the probabilistic representation model, this is done by constructing a discrete model (i.e., by generating a small number of samples) in accordance with a given probability distribution contained in the probabilistic representation.

The current theory also proposes that *symmetry inferences* are central to syllogistic reasoning performance. The symmetry inference is prevalent not only in syllogisms, but also in other areas. For example, a conditional "If *X* then *Y*" is often interpreted as if it also means that "If *not-X* then *not-Y*" or "If *Y* then *X*" at the same time (e.g., Geis & Zwicky, 1971; Staudenmayer, 1975). A logic-based account for this inference is that the conditional "$X \rightarrow Y$" is prone to be interpreted as a *biconditional* "$X \leftrightarrow Y$" (e.g., Johnson-Laird & Byrne, 1991). Similarly, if one is told that the probability of a woman who has breast cancer receiving a positive mammography is 80%, then one is apt to infer that the probability that a woman who tested positive actually has breast cancer is also about 80%, even if the answer clearly violates the Bayesian norm (Eddy, 1982; Gigerenzer & Hoffrage, 1995; Tversky & Kahneman, 1980). Many researchers have attributed this type of error to the *inverse fallacy*, a tendency to confuse a given conditional probability P(*symptom* | *disease*) with the inverse conditional probability, P(*disease* | *symptom*), that is to be judged (Braine, Connell, Freitag, & O'Brien, 1990; Gavanski & Hui, 1992; Hammerton, 1973; Koehler, 1996; Macchi, 1995; Villejoubert & Mandel, 2002; Wolfe, 1995). These are all examples of the symmetry inference.

One of the reasons why symmetry inference is important for a comprehensive theory of thinking is that this mode of inference has been argued to be distinctively human. Nonhuman animals such as chimps (Dugdale & Lowe, 1990, 2000), find symmetry inferences extremely difficult (e.g., D'Amato, Salmon, Loukas, & Tomie, 1985; Sidman et al., 1982). Many researchers have pointed to the relevance of symmetry to language processing (e.g., Dugdale & Lowe, 1990; Horne & Lowe, 1996; Oaksford, 2008) or to creativity (Hattori, 2008). The fundamental ability to perform symmetry inferences may be constrained by phylogenetic factors, and is closely related to other areas of cognition such as language and creativity that are only found in humans. Thus, appearance of symmetry inferences in syllogistic reasoning may be a reflection of our common cognitive architecture.

A theory with probabilistic representations may afford an insight into the nature of symmetry inferences. Hattori and Nishida (2009) hypothesized that people tend to regard two target classes of objects or events as almost equal in size (see Fig. 1). For example, when we think of a disease (e.g., breast cancer) and its symptoms (e.g., a positive mammography), we assume that the sizes of two target sets, one for the disease and the other for symptoms, are roughly the same. This default assumption results in the inverse fallacy. It is reasonable to assume that the target events have a similar probability, unless we know this is not the case (e.g., showing many false positives for a rare disease), because we then gain some information about the credibility of the test. Thus equating the sizes of two target sets (i.e., a *set-size balancing principle*[2]) is a reasonable model of ignorance and the simplest assumption. This principle is known to be maintained in other areas of human thinking, including causal induction (Hattori & Oaksford, 2007) and reasoning in the Wason selection task (Hattori, 2002). Therefore the current theory can reveal an important new link between deductive reasoning and other areas of thinking.

I now briefly introduce the syllogistic reasoning task and some of the terminology required to understand the literature and the current theory before turning to review of previous studies. I follow the orthodox Aristotelian classification in this paper, although there are several different forms of notation used in the psychological literature (see also Appendix A). Syllogisms are constructed with two premises and one conclusion. Each statement is one of four forms called *moods*. Traditionally, these are labeled A, I, E, and O:

[2] It is called the *equiprobability assumption* in Hattori and Oaksford (2007) and Hattori and Nishida (2009). However, some researchers use the same term with the different meaning that each individual possibility has the same probability (e.g., Johnson-Laird et al., 1999; Lecoutre, 1992). To avoid confusion, I adopt a different name here.

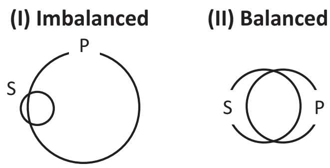


Fig. 1. Probabilistic structures.

A: All *X* are *Y*
I: Some *X* are *Y*
E: No *X* are *Y*
O: Some *X* are not *Y*

The subject (S) and predicate (P) in the conclusion are called *end terms*, and a term that does not appear in the conclusion is called a *middle term* (M). The two terms *X* and *Y* in the first premise correspond to P and M, or M and P, respectively; likewise, *X* and *Y* in the second premise correspond to S and M, or M and S, respectively. As each premise has two possibilities, there are four possibilities for the positions of end and middle terms, which are called *figures*, as shown in Fig. 2 (see also Appendix A). Because each of two premises can be one of the four moods, and there are four possibilities regarding the position of terms, there are $4 \times 4 \times 4 = 64$ possible types of premises for logical syllogisms. These are expressed by a set of three symbols, such as AA1, indicating the mood of the first premise, the mood of the second premise, and the figure (1–4, see Fig. 2). Of these 64 syllogisms, only 19 have a logically valid conclusion that can be expressed in terms of A, I, E, and O, as shown in Table 1 (the validity of a syllogism is somewhat controversial, see Appendix A).

Symmetry inferences in syllogistic reasoning were first articulated by Chapman and Chapman (1959), which is now called *illicit conversion*, although the idea dates back to the early work of Wilkins (1928) and Sells (1936). Their seminal (albeit ambiguous) ideas, on which the current study relies heavily, seemed to account for the trend of people's major responses to syllogisms, as well as the atmosphere hypothesis (Woodworth & Sells, 1935). However, it was not until the 1970s that a comprehensive theory appeared. Ceraso and Provitesa (1971) first proposed the concept of the *indeterminacy* of representations with regard to Euler circle representations of syllogisms. This concept is relevant to some major theories in this area. They noted that premises in forms A, I, and O are ambiguous in terms of specifying the set relations between the terms in the premise. That is, "All *X* are *Y*" (A) is compatible with two possible set relations, $X = Y$ and $X \subset Y$ (*X* is included in *Y*), which correspond to Euler circles D0 and D1, respectively, in Fig. 3. Developing this idea, Erickson (1974) proposed a *set analysis theory*, which Guyote and Sternberg (1981) later developed into a much specific model called the *transitive-chain theory*, which is a hybrid model of logic and probability. Each premise has one or more possible representations in terms of set relations. This is the main reason why the combined representation for two premises of a syllogism always has alternatives. To handle this indeterminacy, Guyote and Sternberg (1981) introduced probabilities into the model, and established representational priorities to derive a logical conclusion. Their highly complicated parameterized model was the first to qualitatively predict participants' response patterns, and exhibited a good fit to the available data.

(1)	(2)	(3)	(4)
M – P	P – M	M – P	P – M
S – M	S – M	M – S	M – S
S – P	S – P	S – P	S – P

Fig. 2. Syllogistic figures.

Table 1
All types of syllogism with their valid conclusions, and predictions by the mental models theory.

No	Type	Valid conclusion		MMT			
		Aristotle	J-L	#MM	1st	2nd	3rd
1	AA1	A	A	1	A,A′		
2	AA2	–	–	2	A,A′	N	
3	AA3	I	I,I′	1	A,A′		
4	AA4	I	A′	1	A,A′		
5	AI1	I	I,I′	1	I,I′		
6	AI2	–	–	2	I,I′	N	
7	AI3	I	I,I′	1	I,I′		
8	AI4	–	–	2	I,I′	N	
9	IA1	–	–	2	I,I′	N	
10	IA2	–	–	2	I,I′	N	
11	IA3	I	I,I′	1	I,I′		
12	IA4	I	I,I′	1	I,I′		
13	AE1	–	O′	3	E,E′	O,N	O′
14	AE2	E	E,E′	1	E,E′		
15	AE3	–	O′	3	E,E′	O,N	O′
16	AE4	E	E,E′	1	E,E′		
17	EA1	E	E,E′	1	E,E′		
18	EA2	E	E,E′	1	E,E′		
19	EA3	O	O	3	E,E′	O′,N	O
20	EA4	O	O	3	E,E′	O′,N	O
21	AO1	–	–	2	O,O′	N	
22	AO2	O	O	2	O′	O,N	
23	AO3	–	O′	2	O	O′,N	
24	AO4	–	–	2	O,O′	N	
25	OA1	–	–	2	O,O′	N	
26	OA2	–	O′	2	O	O′,N	
27	OA3	O	O	2	O′	O,N	
28	OA4	–	–	2	O,O′	N	
29	II1	–	–	2	I,I′	N	
30	II2	–	–	2	I,I′	N	
31	II3	–	–	2	I,I′	N	
32	II4	–	–	2	I,I′	N	
33	IE1	–	O′	3	E,E′	O,N	O′
34	IE2	–	O′	3	E,E′	O,N	O′
35	IE3	–	O′	3	E,E′	O,N	O′
36	IE4	–	O′	3	E,E′	O,N	O′
37	EI1	O	O	3	E,E′	O′,N	O
38	EI2	O	O	3	E,E′	O′,N	O
39	EI3	O	O	3	E,E′	O′,N	O
40	EI4	O	O	3	E,E′	O′,N	O
41	IO1	–	–	2	O,O′	N	
42	IO2	–	–	2	O,O′	N	
43	IO3	–	–	2	O,O′	N	
44	IO4	–	–	2	O,O′	N	
45	OI1	–	–	2	O,O′	N	
46	OI2	–	–	2	O,O′	N	
47	OI3	–	–	2	O,O′	N	
48	OI4	–	–	2	O,O′	N	
49	EE1	–	–	2	E,E′	N	
50	EE2	–	–	2	E,E′	N	
51	EE3	–	–	2	E,E′	N	
52	EE4	–	–	2	E,E′	N	
53	EO1	–	–	2	E,E′	N	
54	EO2	–	–	2	E,E′	N	
55	EO3	–	–	2	E,E′	N	
56	EO4	–	–	2	E,E′	N	
57	OE1	–	–	2	E,E′	N	
58	OE2	–	–	2	E,E′	N	
59	OE3	–	–	2	E,E′	N	
60	OE4	–	–	2	E,E′	N	
61	OO1	–	–	2	O,O′	N	
62	OO2	–	–	2	O,O′	N	
63	OO3	–	–	2	O,O′	N	
64	OO4	–	–	2	O,O′	N	

Note. The prime symbol indicates that the order of terms is converted (i.e., P–S instead of S–P). The "–" sign indicates there is "No valid conclusion." N indicates the model predicts the "No valid conclusion" response. "J-L" indicates Johnson-Laird's definition.

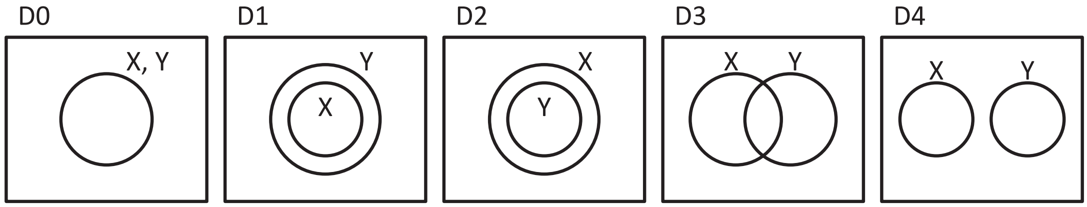


Fig. 3. Euler circles representing the relationships between two sets, *X* and *Y*.

The idea of incorporating probability into a model of syllogistic reasoning, which is normatively non-probabilistic, was later followed by the *probabilistic heuristics model* (Chater & Oaksford, 1999), albeit this placed issues about the nature of mental representations to one side. The indeterminate nature of mental representations was also a key idea in the *mental models theory* (Johnson-Laird & Bara, 1984; Johnson-Laird & Steedman, 1978), but this model could not elicit quantitative predictions. As such, the probability heuristics model and the mental model theory appear to be mutually exclusive. The current theory based on probabilistic representation is an intersection of these theories, intended to provide a new integrated theory.

2. A theory of probabilistic representation

In this section, I present a model for syllogistic reasoning based on probabilistic representations.[3] This theory, unlike previous probabilistic approaches, predicts people's behavior in syllogisms based on internal representations. The model's two major assumptions are (1) that *probabilistic representations* are constructed and (2) that inferences are based on *individual-based representations*. The model assumes that people reason by picturing a "probable" state of affairs drawn from the premises of a syllogism instead of an exhaustive logical scrutiny of propositions in the premises. The model also includes three key conceptions: (1) minimal constraints on logical relations, (2) a small number of samples, and (3) the informativeness of statements. Each conception corresponds to one step of the model, as detailed below.

2.1. Outline of the model

First, I briefly illustrate the conception of *probabilistic inference* that was first introduced by Chapman and Chapman (1959). I then attempt to reformulate this concept.

2.1.1. Probabilistic inference

Here, I introduce a sketch of a probabilistic inference, the heart of the current theory. According to Chapman and Chapman (1959), people regard the middle term of a syllogism as a common quality or effect: people reason that "things that have common qualities or effects are likely to be the same kinds of things, but things that lack common qualities or effects are not likely to be the same" (p. 225). For example, given the premises that "Some Practitioners are Mediators" and "Some Sophists are *not* Mediators" (IO2), Sophists and Practitioners are not likely to be considered the same kinds of people, and thus "Some Sophists are not Practitioners" (O) is falsely concluded by probabilistic inference.

I now provide a more formal explanation of my interpretation of this idea. Logically speaking, there can be some individuals who are both Sophists (S) and Practitioners (P), but not Mediators (M); in fact, all individuals can be so (i.e., both S and P, but not M). Therefore, the O-conclusion (shown above) must be rejected. However, people seem to infer that the probability that an individual who is S but not M (warranted by the second premise) also happens to be P is very small; thus, the O-conclusion is not suppressed. This view is justified as follows by the *rarity assumption* (Oaksford & Chater, 1994). As the probability of S is small (i.e., rare), the probability that some arbitrary individual is S but not M is smaller. Moreover, the probability of P is small (i.e., rare). Therefore, the probability that an individual is S but not M and also P would be much smaller. This means that the conclusion is hardly refuted, or the conclusion has some probability of being endorsed. This sketch instantiates how our (deductive) inference is affected by probabilistic information. It shows how individual heuristics are incorporated in a process of deduction, and also how an original idea of probabilistic inference is linked to mathematical probabilities.

2.1.2. Model assumptions and steps

The model has three major assumptions:

(1) The assumption of *probabilistic representations*: Models that represent a state of affairs include probabilistic information, which is sometimes logically incorrect.
(2) The assumption of *individual-based representations*: "Logical" (i.e., discrete) inferences are made based on a finite number of individual elements.
(3) The assumption of *possibility*: A logically possible (not necessary) conclusion is derived based on the individual-based representation.

According to the current theory, probabilistic information that is essentially irrelevant to descriptions of the logical status or to logical inferences will (inevitably) affect our deductive reasoning. This is because the construction of probabilistic representations precedes individual-based representations. Contrary to assumption (1), most psychological theories on syllogistic reasoning, as well as the ordinary Euler circle representation, do not distinguish between two diagrams that are topologically identical.

The second point of the model is the process of assessing logical relationships based on a limited number of individuals. The model inherits the idea that a class is represented by limited number of individuals from the mental model theory. The new model assumes that we have difficulty thinking about many elements at the same time because of working memory limitations. A logical statement that is not inconsistent with the current finite sample is assumed to be derived. I introduce the concept of random sampling to connect these two assumptions.

The third point of the model is that we derive a statement as a "logical" conclusion if the statement tested is consistent with the individual-based model that indicates a possibility (not the necessity) of given premises (Evans, Handley, Harper, & Johnson-Laird, 1999). The test is conducted sequentially, and the informativeness of statements affects the process. The model process in drawing a conclusion from the premises of a syllogism is assumed to consist of three steps:

[3] The model code (implemented in R, ver. 3.1.0 or later) together with all data used in this article is available online (Hattori, 2016).

(1) Constructing a *probability prototype model* (*PPM*): Given a syllogism, people construct a representative model with minimal logical constraints other than those given by the two premises. This model includes probabilistic information about the occurrence of events.
(2) Constructing a *sample mental model* (*SMM*): Generating a small number of data in accordance with the PPM, people construct an SMM.
(3) Generating a logical conclusion: People sequentially (in descending order of informativeness) examine which of the quantified statements is consistent with the SMM, and the first one that the model fulfills is chosen as the conclusion.

2.1.3. An illustration

An outline of the model is schematized in Fig. 4. I now provide a specific explanation of how the model behaves. In the case of AI2, for example, the two premises are as follows:

(1) All P are M
(2) Some S are M

The first premise causes people to assume the relationship between P and M is like D1 in Fig. 3, and the second premise causes them to assume the relationship between S and M is like D3. In fact, the first premise is compatible with D0 and D1, and the second premise is compatible with D0, D1, D2, and D3, but I regard D1 and D3 as "standard" diagrams for each premise. In the first step, combining two diagrams with minimal constraints, people construct a PPM as shown in Step 1 of Fig. 4. This model assigns probabilities for each area corresponding to $2 \times 2 \times 2 = 8$ possible combinations of the truth values for S, M, and P.

The point here is that a probabilistic representation (PPM) is constructed in advance before an individual-based mental model (SMM). In this point, the probabilistic representation theory is distinctive from other theories, including the latest extension of mental model theory (Khemlani et al., 2015), as mentioned in

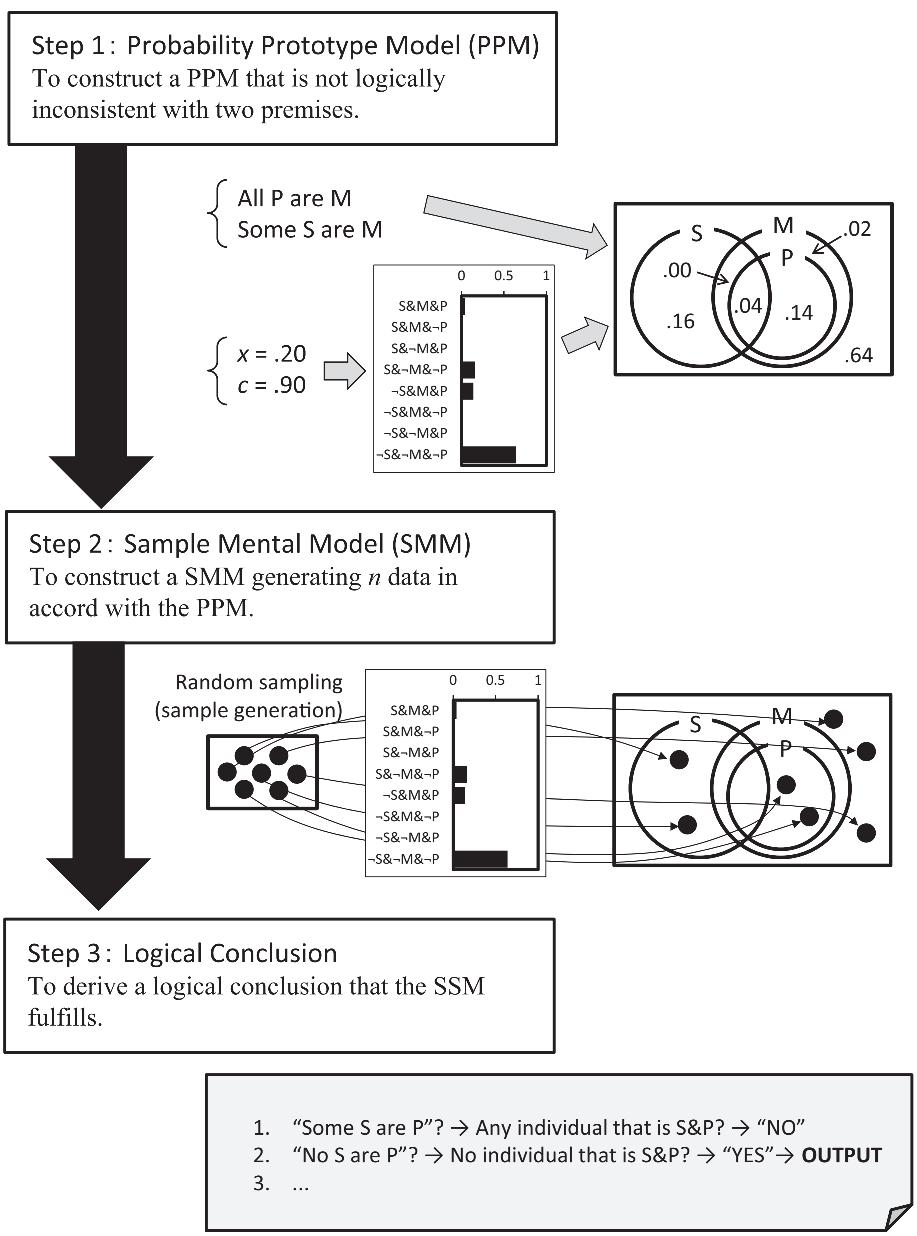


Fig. 4. Outline of the model: in the case of AI2.

Section 1. The probability distribution that determines the probability value of each Euler circle area is defined by two parameters (i.e., x and c) detailed later (Section 2.2.2) and the structural constraints given by logic.

In the second step, a comparatively small number of elements (e.g., seven) within an SMM are randomly generated according to the probability distribution defined in the PPM. So the logical status of each element in a discrete mental model (i.e., SMM) is derived from a random assignment based on a continuous probabilistic representation (i.e., PPM). Reasoning processes based on elements in a probable state of affairs are realized by Steps 1 and 2: hard-coded standard (not necessarily logical) relationships among terms in the premises (Step 1) and probabilistic sampling (Step 2).

Finally, people test the logical relationships of the SMM in terms of S and P, and output their response. In this case, the first inspection is "Some S are P" (I), but the particular SMM exemplified in Fig. 4 happens to be inconsistent with this statement, and the next, "No S are P" (E), is verified. Thus, the final statement is output as a conclusion. In this procedure, the order of tests greatly affects the model's performance. The order is determined according to the min-heuristic and the max-heuristic identified by Chater and Oaksford (1999). This example outlines the behavior of the model. I now describe the three steps of the model in detail.

2.2. Step 1: Constructing a probability prototype model

When engaging in syllogistic reasoning, we have to interpret and represent two premises, and then integrate these representations into a single representation. As I mentioned in Section 1, the problem of indeterminacy emerges in this encoding stage. I assume this encoding is conducted following a law of simplicity: (1) only one representative representation is adopted for each premise and (2) only one minimally restricted probabilistic representation is formed by combining two representations for premises. Here, I explain the model's specifications using Euler diagrams for the sake of convenience, but note that the graphical representation is not essential for the modeling, and any other equivalent representations, including some mental tokens (that must be equipped with probabilities) could perform the same function.

2.2.1. Euler circle representations

Each quantifier, A, I, E, and O, corresponds to one or some Euler circle representations, known as the Gergonne relations (Faris, 1955). An A-statement is compatible with Euler representations D0 and D1 in Fig. 3. Likewise, I-, E-, and O-statements are compatible with D0, D1, D2, and D3; with D4; and with D2, D3, and D4, respectively. Let me define a *standard* Euler representation for each quantifier from the view of minimal logical constraints. Probabilistic constraints for D0 are $P(X, \bar{Y}) = 0$ and $P(\bar{X}, Y) = 0$, and for D1 are $P(X, \bar{Y}) = 0$ and $P(\bar{X}, Y) > 0$. The difference between D0 and D1 is the constraint on the value of $P(\bar{X}, Y)$. D1 can be seen as having less of a constraint than D0, because $P(\bar{X}, Y)$ of D1 (i.e., a value greater than 0 and smaller than or equal to 1) has a much higher degree of freedom than that of D2 (i.e., 0, which is a particular point on a number line). The Euler circle with the highest freedom among possible variations under the probabilistic constraints imposed by a premise is called the *standard* Euler representation of the premise. According to this definition, the standard representation of I is D3, because D3 is the least constrained diagram among the four. Likewise, the standard representations of E and O are D4 and D3, respectively. The idea of this standard representation is based on the minimization of logical constraints, or maximization of independence. Note that Stenning and Oberlander's (1995) purely logical detailed analyses provided similar results.

To draw a conclusion from two premises, the Euler circle representations for the premises should be combined into one. In some cases, however, several possibilities emerge at this stage. In such cases, the least logically constrained representation is again adopted. For example, as shown in Fig. 5, three Euler representations are possible for AI2 when combining D1 for the first premise and D3 for the second premise. Among the three, we choose Fig. 5-III as the standard representation, in which independence between S and P conditioned by M can be assumed. That is, given that the first premise imposes constraint $P(\mathrm{M}|\mathrm{P}) = 1$, and the second premise requires that $P(\mathrm{S}, \mathrm{M}) > 0$, the maximum freedom is allowed between the end terms by assuming their conditional independence (Chater & Oaksford, 1999; Pearl, 1988) given the middle term, $P(\mathrm{S}, \mathrm{P}|\mathrm{M}) = P(\mathrm{S}|\mathrm{M})P(\mathrm{P}|\mathrm{M})$. The conditional independence is important for the probability heuristic models (Chater & Oaksford, 1999) as it affects the *p*-validity of conclusions, and is thus incorporated into the current theory although it does not directly use *p*-validity. The standard Euler circle representations for all syllogisms are shown in Fig. 6.

2.2.2. Assigning probabilities to Euler circles: two parameters

Next, we assign probabilities to each of the subsets of the Euler circles. For this procedure, I again assume a law of simplicity. A combination of three sets, S, M, and P, yields a maximum of $2^3 = 8$ subsets, indicated by Arabic numerals 1–8 in Fig. 6 (see the note in Table 2 for the correspondence between each numeral and its logical status). I introduce the simplest policy that assumes all terms have the same probability (expressed by a parameter x), unless this violates the logical constraints of the premises, and they retain maximal independence. First, the probability of the middle term $P(\mathrm{M})$ is assumed to be expressed by x:

$$x = P(\mathrm{M}). \quad (1)$$

Unless the premise is A, S and P are allowed to have the same probability as M, and we can assume

$$P(\mathrm{S}) = x, \quad (2)$$

$$P(\mathrm{P}) = x. \quad (3)$$

When the premise is A, the probability is defined by a *coverage* parameter c ($0 < c \leqslant 1$). If the premise is "All X are M" (X stands for S or P),

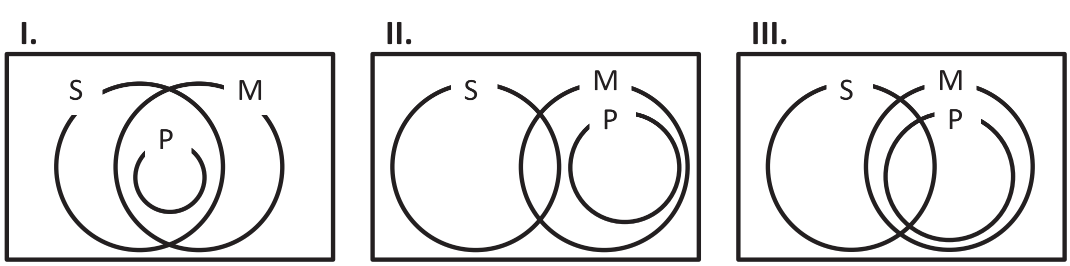


Fig. 5. Three possible combined Euler circles for AI2 when the two premises are D1 and D3 in Fig. 3. The probability prototype model (PPM) of AI2 is assumed to be III, because it maximizes the independence.

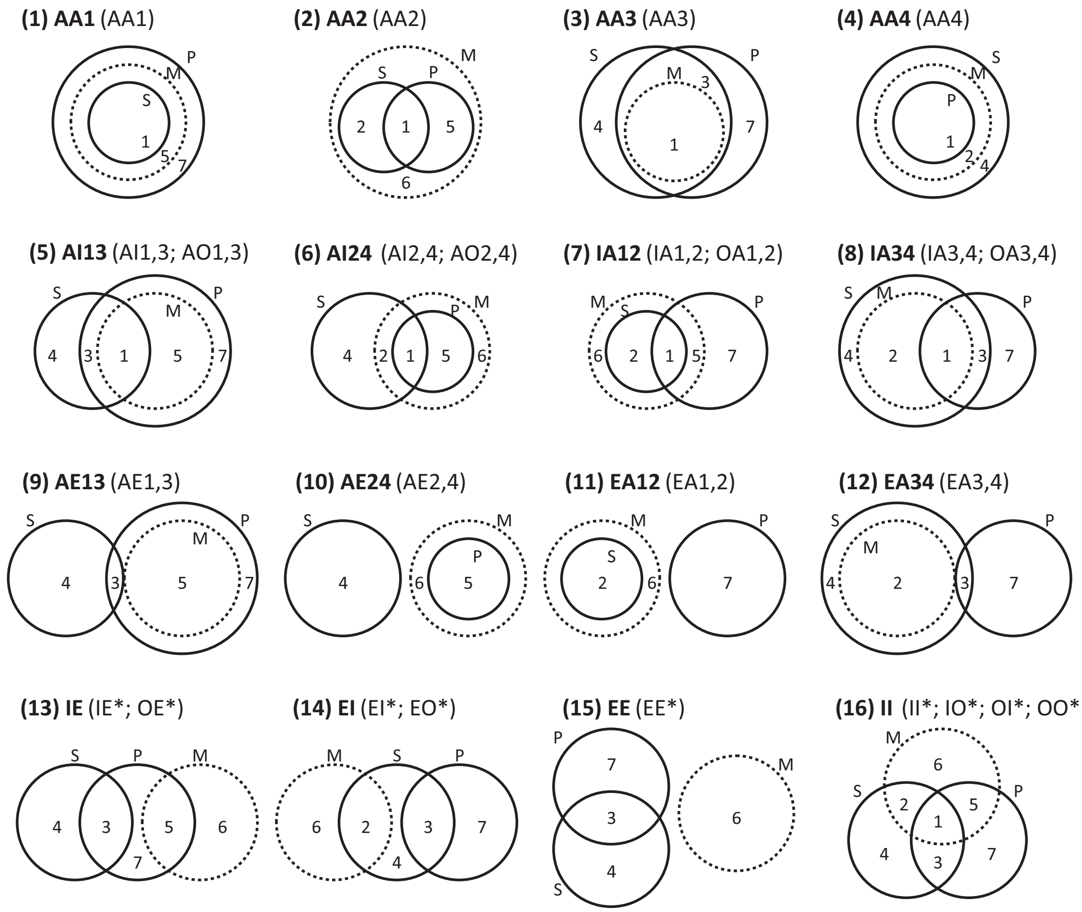


Fig. 6. PPM for each syllogism expressed by the Euler diagram. Each model corresponds to the syllogisms listed in parentheses. Asterisks in parentheses indicate the wildcard (e.g., IE∗ indicates IE1,2,3,4). S, M, and P indicate syllogistic terms and Arabic numerals indicate the logical status of the corresponding areas (e.g., 1 indicates S and M and P; see the note to Table 2).

$$P(X) = cx. \tag{4}$$

Alternatively, if the premise is "All M are *X*,"

$$P(X) = 1 - c(1 - x). \tag{5}$$

The coverage parameter means that the overlap between *X* and M grows as *c* approaches 1, and *X* completely coincides with M when $c = 1$.

Second, joint probabilities for S, M, and P are defined as follows. If the premise is A ("All *X* are *Y*"), the logical constraint implies the following relationship:

$$P(X, \bar{Y}) = 0. \tag{6}$$

Similarly, if a premise is E ("No *X* are *Y*"), the following relationship is implied:

$$P(X, Y) = 0. \tag{7}$$

The probabilistic interpretations of I ("Some *X* are *Y*") and O ("Some *X* are not *Y*") would be $P(X, Y) > 0$ and $P(X, \bar{Y}) > 0$, respectively, but these conditions do not constrain any assigned probability values. Consequently, the two parameters, *x* and *c*, uniquely define probabilistic representations for all syllogisms.

2.2.3. An illustration

Incorporating Eqs. (1)–(3) as assumptions and Eqs. (4)–(7) as constraints imposed by logic, and allowing maximal independence among terms, the joint probability distribution for S, M, and P can be defined. Table 2 shows the joint probability distributions for all types of syllogisms defined by this procedure. I now explain the procedure of probability allocation using specific examples. In the case of AA2, as the first premise is "All P are M," $P(\mathrm{M}) = x$ and $P(\mathrm{P}) = cx$ from Eqs. (1) and (4). According to the conditional independence between S and P, $P(\mathrm{S}, \mathrm{P}|\mathrm{M}) = P(\mathrm{S}|\mathrm{M})P(\mathrm{P}|\mathrm{M})$. Therefore, $P_1 = P(\mathrm{S}, \mathrm{M}, \mathrm{P}) = \frac{P(\mathrm{S},\mathrm{M})P(\mathrm{P},\mathrm{M})}{P(\mathrm{M})} = \frac{P(\mathrm{S})P(\mathrm{P})}{P(\mathrm{M})} = c^2x$, where P_1 denotes $P(\mathrm{S}, \mathrm{M}, \mathrm{P})$. $P_2, \ldots, P_8$ are also defined in Table 2. The PPM for AA2 (Fig. 6-2) indicates that $P_2 = P(\mathrm{S}) - P_1 = cx - c^2x = c(1 - c)x$, and $P_3 = 0$. Then, P_4, P_6, P_7, and P_8 are automatically derived as described in the note in Table 2.

The next example is EE1. In this case, the PPM (Fig. 6-15) indicates that $P_1 = P_2 = P_5 = 0$. As the conditional independence is also maintained on the outside of M, $P(\mathrm{S}, \mathrm{P}|\bar{\mathrm{M}}) = P(\mathrm{S}|\bar{\mathrm{M}})P(\mathrm{P}|\bar{\mathrm{M}})$. Therefore, $P_3 = \frac{(P_3+P_4)(P_3+P_7)}{1-x} = \frac{x^2}{1-x}$, and the other probabilities (P_4, P_6, P_7, and P_8) are fixed.

2.3. Step 2: Constructing a sample mental model

In this step, an *individual-based mental representation* is constructed using instances generated by a random sampling procedure based on the probabilities defined by the PPM. Theoretically speaking, if the number of instances generated approaches to infinity, the logical relationship that the mental model satisfies coincides with its original PPM's logical relationship (i.e., the one satisfied by the corresponding Euler circle shown in Fig. 6). That is to say, "AE24" and "EA12" in Fig. 6 entail both E and O; "AA1" entails both A and I; and the others entail both I and O.

However, if the number of instances is finite, and particularly if it is small, the logical relationship in a PPM is not guaranteed in the corresponding SMM. This is the point of the SMM. For example, in

Table 2
Probabilities of all subsets in each PPM.

No	Name	Type	$P(\mathrm{S})$	$P(\mathrm{P})$	P_1	P_2	P_3	P_5
					$P(S,M,P)$	$P(S,M,\bar{P})$	$P(S,\bar{M},P)$	$P(\bar{S},M,P)$
1	AA1	AA1	cx	y	cx	0	0	$\bar{c}x$
2	AA2	AA2	cx	cx	c^2x	$c\bar{c}x$	0	$c\bar{c}x$
3	AA3	AA3	y	y	x	0	$\bar{c}^2\bar{x}$	0
4	AA4	AA4	y	cx	cx	$\bar{c}x$	0	0
5	AI13	AI1,3; AO1,3	x	y	x^2	0	$\bar{c}x\bar{x}$	$x\bar{x}$
6	AI24	AI2,4; AO2,4	x	cx	cx^2	$\bar{c}x^2$	0	$cx\bar{x}$
7	IA12	IA1,2; OA1,2	cx	x	cx^2	$cx\bar{x}$	0	$\bar{c}x^2$
8	IA34	IA3,4; OA3,4	y	x	x^2	$x\bar{x}$	$\bar{c}x\bar{x}$	0
9	AE13	AE1,3	x	y	0	0	$\bar{c}x\bar{x}$	x
10	AE24	AE2,4	x	cx	0	0	0	cx
11	EA12	EA1,2	cx	x	0	cx	0	0
12	EA34	EA3,4	y	x	0	x	$\bar{c}x\bar{x}$	0
13	IE	IE*; OE*	x	x	0	0	x^2	x^2
14	EI	EI*; EO*	x	x	0	x^2	x^2	0
15	EE	EE*	x	x	0	0	$x^2/\bar{x}$	0
16	II	II*; IO*; OI*; OO*	x	x	x^3	$x^2\bar{x}$	$x^2\bar{x}$	$x^2\bar{x}$

Note. Parameters x and c indicate $P(\mathrm{M})$ and degree of coverage, respectively (see text in detail). In this table, $\bar{x}$ and $\bar{c}$ stand for $1-x$ and $1-c$, respectively; and $y = x + \bar{c}\bar{x}$. Probabilities of other areas can be derived using values given in this table as follows:
$P_4 = P(S,\bar{M},\bar{P}) = P(S) - P_1 - P_2 - P_3$.
$P_6 = P(\bar{S},M,\bar{P}) = P(M) - P_1 - P_2 - P_5$.
$P_7 = P(\bar{S},\bar{M},P) = P(S) - P_1 - P_3 - P_5$.
$P_8 = P(\bar{S},\bar{M},\bar{P}) = 1 - P_1 - P_2 - P_3 - P_4 - P_5 - P_6$.

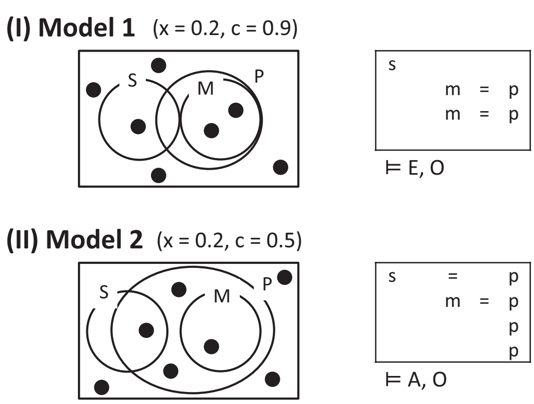


Fig. 7. Two examples of sample mental models (SMMs) for syllogism AE1. Parameters alter the probability of each instance being generated. Equivalent representations expressed by the notation of the mental model theory are shown in the right portions.

the case of "AE13" in Fig. 6, if c is nearly equal to 1, then M and P largely overlap, and the probabilities of areas 3 and 7 are small. This makes samples of conjunctive instances for S and P hard to obtain, an I-conclusion becomes hard to derive, and an O- or an E-conclusion is promoted. The sample size is assumed to be seven, which is the approximate size of our working memory, unless otherwise stated (as in Section 4.7). I now illustrate this mechanism.

For AE1, if we assume $x = 0.2$ and $c = 0.9$, the probabilities of areas 3, 4, 5, 7, and 8 (see the note in Table 2) would be 0.06, 0.14, 0.20, 0.02, and 0.58, respectively, according to Table 2. As a result of sampling seven instances based on these probabilities, assume we now have one instance in area 4, two in area 5, and four in area 8, as shown in Fig. 7-I. This SMM can be represented in the notation of the mental model theory, as shown in the right panel of Fig. 7-I.[4] In this model, as there is no instance that satisfies both S and P, an E- or an O-conclusion will be derived. Which of these is derived depends on the procedure defined in the next step, where a logical conclusion is generated (detailed in Section 2.4).

Alternatively, if we assume that $x = 0.2$ and $c = 0.5$ for the same syllogism, AE1, the probabilities of areas 3, 4, 5, 7, and 8 would be 0.08,0.12,0.20, 0.32, and 0.28, respectively, according to Table 2. Suppose that one, one, two, and three instances are obtained in areas 3, 5, 7, and 8, respectively, based on random sampling, resulting in the SMM shown in Fig. 7-II. In this case, an A- or I-conclusion, different from the previous case, will be derived.

2.4. Step 3: Generating a logical conclusion and its probability

Once an SMM has been constructed, the next step is to derive a logical conclusion by examining the logical relationships between S and P according to the samples in the model. In most cases, however, multiple statements would be compatible with an instance-based model. Logically speaking, A implies I, and E implies O. Therefore, both A- and I-conclusions, and both E- and O-conclusions, are available in some cases. Moreover, in many cases, I- and O-conclusions are not inconsistent with each other. Therefore, the procedure for selecting a conclusion from multiple candidates must be clarified to make a prediction of participants' behavior.

2.4.1. Order of the tests

I assume an SMM is sequentially tested according to the informativeness of the syllogistic statements analyzed by Chater and Oaksford (1999). The model incorporates two important heuristics for forming the output from the probability heuristics model: the *min-heuristic* and the *max-heuristic*. The min-heuristic is used to define the order of testing at the final stage. This is expected to realize an efficient procedure in which more probable candidates would be tested earlier. It is intuitive that uninformative statements can generally provide only a weak argument. In this sense, the informativeness of syllogistic statements restricts the type of valid conclusions. The min-heuristic in the probability heuristics model is used to pick the least informative premise of the two as a type of conclusion candidate. Chater and Oaksford (1999) revealed that the order of informativeness of statements under

[4] The representation is not exactly the same as Johnson-Laird's, because in this paper I simplify the expression (without altering the number of models) as long as it does not affect the conclusion. I have tried not to use the notation "0s" or "(s)" that appeared in their earlier papers, which indicate that it is uncertain whether or not the relevant individual exists.

the rarity assumption is $I(\mathrm{A}) > I(\mathrm{I}) > I(\mathrm{E}) > I(\mathrm{O})$, and this order is adopted for testing in the probabilistic representation model.

In this model, the max-heuristic is applied at the very last moment to output the conclusion. After determining a provisional candidate conclusion in the above process, the probability of adopting it as the final output (i.e., the degree of confidence in this candidate as an actual conclusion) is defined by the type of quantifiers of the premises according to the max-heuristic. The max-heuristic in the probability heuristics model defines the degree of confidence in the conclusion in proportion to the informativeness of the most informative premise. Following this idea, in the probabilistic representation model, the degree of confidence is set by semi-fixed parameters (i.e., u_A, u_I, u_E, and u_O) defined by the most informative premise (i.e., A, I, E, or O). In practice, the overall algorithm of the model is as follows (see Fig. 8):

(1) Choose the least informative statement of the two premises (i.e., the min-heuristic), and set this as the target mood of the target statement to be tested.
(2) The given SMM is tested for consistency with the target statement. The criteria for judgment are detailed below. If the statement is validated, output it as a conclusion with a probability defined by the max-heuristic parameters; otherwise, go to (3).
(3) Choose the next most informative mood as the target to be tested. If the mood to be tested reaches O, then go back to A. If all four moods are exhausted, output N (i.e., "no valid conclusion") as the conclusion. Otherwise, return to (2).

In a series of consistency tests, if a candidate conclusion is necessary for the targeted SMM, it is adopted as the conclusion. Put another way, the criteria for testing the consistency of the SMM are as follows:

(1) An **A**-conclusion is derived if there is *not* any individual that is S but *not* P.
(2) An **I**-conclusion is derived if there *is* at least one individual that is both S and P.
(3) An **E**-conclusion is derived if there is *not* any individual that is both S and P.
(4) An **O**-conclusion is derived if there *is* at least one individual that is S but *not* P.

2.4.2. Forming a probability distribution of responses

The model constructed as described above takes two premises of a particular syllogism as input, and produces a conclusion of A, I, E, O, or N as its output. The model simulates, a one-shot inference of a particular person. When the model is iteratively run a considerable number of times, the total pattern of its output is assumed to be a prediction for the result of an experiment that considers a sufficient number of participants. The theoretical distribution of the frequency of responses is analytically derived from a probability distribution defined by a PPM and the number of samples as follows:

An A-statement, "All S are P" ($\mathrm{S}\cap\bar{\mathrm{P}}=\varnothing$), is consistent in an SMM including n samples with the probability that there is no sample in Areas 2 ($\mathrm{S}\cap\mathrm{M}\cap\bar{\mathrm{P}}$) and 4 ($\mathrm{S}\cap\bar{\mathrm{M}}\cap\bar{\mathrm{P}}$) (see Fig. 6). The probability that a sample is in Areas 2 or 4 is P_2+P_4, and the probability that no sample is in either area is $1-P_2-P_4$. As each sample is independent, the probability that n samples are neither in Area 2 nor in Area 4 is $(1-P_2-P_4)^n$.

An E-statement, "No S are P" ($\mathrm{S}\cap\mathrm{P}=\varnothing$), is consistent in an SMM with the probability that there is no sample in Areas 1 ($\mathrm{S}\cap\mathrm{M}\cap\mathrm{P}$) or 3 ($\mathrm{S}\cap\bar{\mathrm{M}}\cap\mathrm{P}$), which is $1-P_1-P_3$. As for the A case, the probability that n samples are neither in Area 1 nor in Area 3 is $(1-P_1-P_3)^n$. An I-statement, "Some S are P" ($\mathrm{S}\cap\mathrm{P}\neq\varnothing$), is consistent if and only if its complement, "No S are P" ($\mathrm{S}\cap\mathrm{P}=\varnothing$), is inconsistent. The probability of the latter is $(1-P_1-P_3)^n$, as shown above, and the probability to be derived is $1-(1-P_1-P_3)^n$. An O-statement, "Some S are not P" ($\mathrm{S}\cap\bar{\mathrm{P}}\neq\varnothing$), is consistent if and only if its complement, "All S are P"

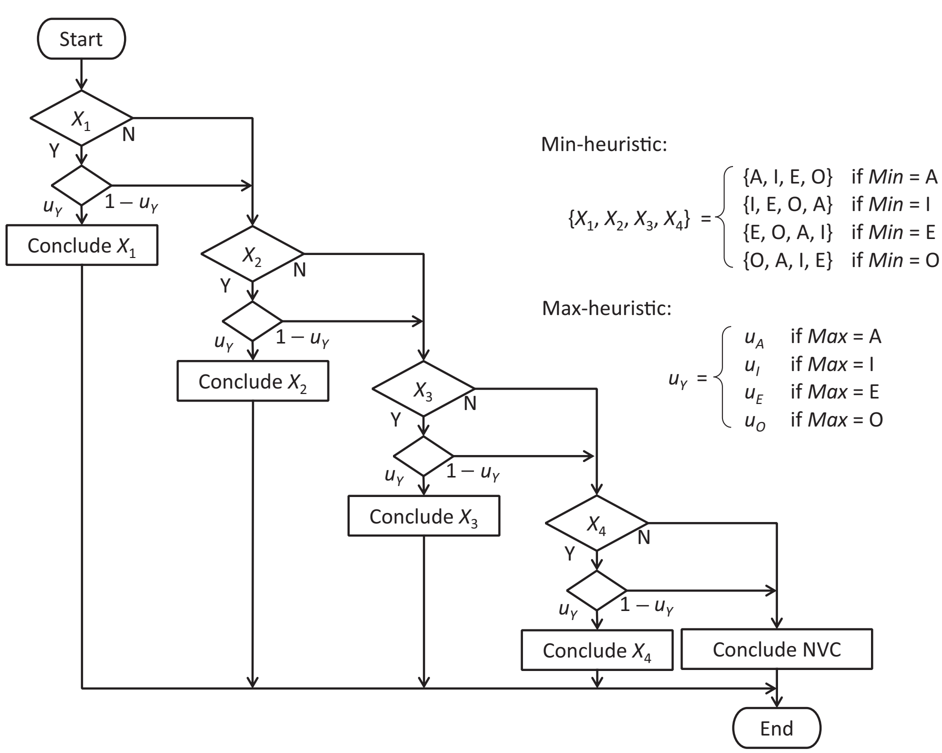


Fig. 8. The process for generating a logical conclusion in the model. Logical conclusions are determined by the order of the tests, the result of each test (depending on the contents of the SMM), and a random effect prescribed by free parameters (u_A, u_I, u_E, and u_O).

($S \cap \bar{P} = \varnothing$), is inconsistent. Therefore, the probability to be derived is $1-(1-P_2-P_4)^n$.

3. Mental representation and probability in other theories

In this section, I review how mental representation and probability, which are the two main issues of this paper, have been dealt with in previous theories of syllogistic reasoning. I discuss the merits and problems of some leading theories that involve at least one of these ideas as their central concept.

3.1. Mental models

The mental model theory (Johnson-Laird, 1983; Johnson-Laird & Bara, 1984; Johnson-Laird & Byrne, 1991; Johnson-Laird & Steedman, 1978) is one of the earliest comprehensive psychological theories of syllogistic reasoning. This theory explains human performance in syllogistic reasoning based on mental representations. The mental model's "crucial characteristics as far as inference is concerned are that a mental model is finite, computable, and contains tokens in relations that represent entities in a specific state of affairs" (Johnson-Laird & Bara, 1984, p. 4). Although this characteristic is basically inherited, the current model is unique in that it includes probabilistic information.[5] For the mental model theory, the number of elements in a mental model is not important, as mental models "represent a set of entities by introducing an *arbitrary* number of elements that denote exemplary members of the set" (Johnson-Laird, 1980, p. 98; italics mine).

As Johnson-Laird and Bara (1984) claim, the mental model theory appears to predict the difficulty of syllogistic reasoning as a function of the number of mental models that must be constructed to derive a logically valid conclusion. For example, EA3 has two premises, E ("No P are M") and A ("All M are S"), and a valid O-conclusion ("Some S are not P"). EA3 is compatible with the two mental models shown in the upper part of Fig. 9, which follows the notation used by Johnson-Laird and Bara (1984).[6] With Model 1 in the upper part of Fig. 9, E- and O-conclusions are compatible; with Model 2, I- and O-conclusions are compatible. However, as E- and I-conclusions are inconsistent with each other, the only statement consistent with both models is the I-conclusion, which becomes the final output (i.e., the correct answer). People who think of Model 1 but fail to construct Model 2 will derive an E-conclusion. As predicted, half of the participants derived an E-conclusion in Experiment 1 conducted by Johnson-Laird and Bara (1984) (and no-one correctly derived an O-conclusion). A task that requires more models to be constructed to reach a correct answer increases the probability of mistakes in the inference process, and would thus be likely to result in failure.

Although the mental model theory is one of the most influential theories at the algorithmic/representational level in Marr's (1982) sense, its explanatory insufficiency is its most critical problem (see, also, Bonatti, 1994; Ford, 1994; Garnham, 1993; Newstead, 1993; Stenning & Oberlander, 1993; Wetherrick, 1993). When we assess the descriptive validity of a model such as the mental model theory, which assumes two steps, we need to know at least two pieces of information for each step: (1) the conditions or procedures to

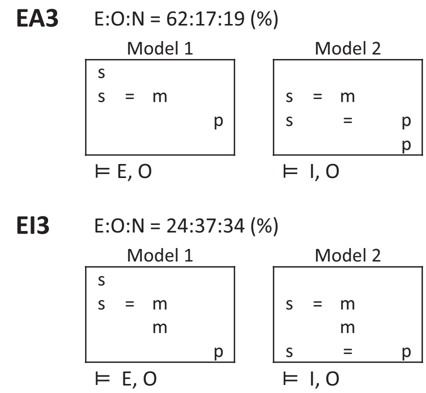


Fig. 9. Mental model representations for syllogisms EA3 (upper) and EI3 (lower). I, E, or O followed by an entailment symbol (⊨) indicate a conclusion consistent with the model, and percentages indicate the mean choice rate in data used in Meta-Analysis 2.

construct mental models like Models 1 and 2 (Fig. 9) and (2) the procedures used to derive conclusions from the constructed models. The mental model theory (Johnson-Laird & Bara, 1984, pp. 35–36), as for the probability heuristics model, assumes that one tries to derive conclusions in decreasing order of informativeness (A > I > E > O), and this assumption satisfactorily specifies procedure (2) above. The mental model theory, however, is not specific enough with (1). What if one constructs Model 2 first? One would then derive an I-conclusion, which is actually a very rare response. The mental model theory does not describe why one particular model is constructed earlier than others when multiple models are available. Although Johnson-Laird and Bara (1984) articulated procedures for constructing alternative models (pp. 36–40) and the number of models needed to derive logically correct conclusions (pp. 52–59), they did not document which model would be sought in what order, and what triggers the next search. Even though, as claimed in their papers, the model is implemented in a computer program, the problem is that the principles of priority in the model construction have never been fully articulated, including in the latest mental model paper on syllogisms (Khemlani & Johnson-Laird, 2012).

The problem comes into focus when we compare one syllogism with another, similar one. EI3 is a two-model syllogism,[7] as shown in the lower panel of Fig. 9. As in the case of EA3, E and O are compatible with Model 1, and I and O are compatible with Model 2. The data (e.g., those used in Meta-Analysis 2), however, indicate a great difference between these two types of syllogism. In EA3, the choice rates for E, O, and N were 62%, 17%, and 19%, respectively, but were 24%, 37%, and 34% in EI3. The mental model theory cannot explain this difference. Many similar instances of discordance can be found among the data for pairs of syllogisms. For example, IA1 and EE2 are two-model syllogisms, neither of which have a valid conclusion (N), but their correct conclusion rates differed greatly (17% vs. 77%).

Indeed, the mental model theory does not even precisely predict the difficulty of syllogisms. According to the mental model theory, the difficulty of syllogisms is determined mainly by the number of models and the figure, as mentioned above. However, my analysis of the same data indicates that the negative correlation between these variables is not particularly strong ($r = -0.61$), and the proportion of correct conclusions for one-, two-, and three-model syllogisms was 18–90%, 17–77%, and 11–50%, respectively. The worst one-model syllogism (AA3: 18%)

[5] Whereas the mental model theory has been extended in various ways, and there is a version that contains probabilistic information (Johnson-Laird et al., 1999; Khemlani, Lotstein, & Johnson-Laird, 2015), none of the basic models represent information on probability or frequency, but instead represent purely logical relationships, as in the case of the transitive-chain theory (see Section 3.2).

[6] The original notation includes tokens like "0s," but here I have simplified the expression. Johnson-Laird and his colleagues treated EA3 as a three-model task, but the third model is not necessary unless we liberate the positions of S and P in the conclusion. Therefore, only two models are shown here.

[7] Johnson-Laird and Bara (1984) actually classified EI3 as a three-model syllogism, as they did EA3. See footnote 4.

was much worse than the best three-model one (EI1: 50%). It is obvious that factors other than the number of models affect the difficulty of syllogisms. Even considering the figural effect, which is another important predictor according to Johnson-Laird and Bara (1984), the result does not alter greatly. In the case of the third figure, for example, the correct conclusion rates for one-, two-, and three-model syllogisms ranged from 21% to 86%, 25% to 77%, and 16% to 36%, respectively.

3.2. Probabilistic approaches

When Chapman and Chapman (1959) first introduced the idea of probability, along with the idea of accepting the converse, into the psychology of syllogistic reasoning, they assumed that people do not think "all but strict deductive reasoning is disallowed" (p. 224), but instead tend to apply strategies that derive a probable conclusion. After 40 years, this idea was refined and realized as several heuristics by Chater and Oaksford (1999). The probability heuristics model includes three heuristics (G1–3) for generating conclusions from given premises and two heuristics (T1–2) for testing the conclusions generated. Among these, the most important are the *min-heuristic* (G1) and the *max-heuristic* (T1). The min-heuristic determines a candidate conclusion. The less informative form of the two premises is chosen as the form of a candidate conclusion. For example, in the case of EA3, Chater and Oaksford's analysis indicates that an E-conclusion will be chosen under the rarity assumption, as the ordering of informativeness is A > I > E > O. The max-heuristic determines the degree of confidence in the conclusion (i.e., the selection rate of the conclusion as an answer). The most informative premise makes the conclusion confident (raises its probability of being selected) to the extent of its informativeness.

Chater and Oaksford (1999) demonstrated that the heuristics they specified could be used as an effectual strategy to derive an appropriate (i.e., *p*-valid, in their terms) conclusion. If a strategy is useful in determining the correct answer, it can be a good heuristic. Moreover, if it is simple and easy to use, it is a highly efficient heuristic. This is a sufficient condition for a model. Their ingenious analysis based on probability theory showed that the heuristics they proposed were justified from an ecological point of view. They also showed, in parallel, that their model provided a good fit to the available data. In other words, the probability heuristics model also satisfied a necessary condition. It is also true, however, that the probability heuristics model is not fully satisfactory as a psychological theory: it has little to say about how mental representations work in the *process* of syllogistic reasoning. It fits the data in hand well, but does not account for the behavior in terms of mechanisms, i.e., the model "does require semantic representations that capture people's understanding of syllogistic premises" (Chater & Oaksford, 1999, p. 236).

With regard to the topic of symmetry inference, both the probability heuristic model and the mental model theory sometimes fail to explain the phenomenon known as conversion. In both AA1 and AA3, for example, the min-heuristic enforces an A-conclusion, and the max-heuristic enhances the confidence of the conclusion in exactly the same way. Neither does the mental model theory distinguish between these types: both AA1 and AA3 are predicted to provide high proportions of correct answers, because both are one-model syllogisms (Johnson-Laird & Bara, 1984).[8] In this regard, the probability heuristics model and the mental model theory are both unsatisfactory.

The transitive-chain theory (Guyote & Sternberg, 1981), which followed from Erickson's (1974, 1978) set analysis theory, is the one of the earliest frameworks that can quantitatively predict response patterns. It preceded the probability heuristics model by about two decades, but was ignored by Chater and Oaksford (1999). The transitive-chain approach is equipped with both mental representations and probability. I should point out, however, that the representation on which these theories are based is purely logical, as is the case for the mental models theory. Continuous parameters, or probabilities, were not introduced into the representations themselves, but into the procedure for handling (i.e., constructing or combining) logical representations. Therefore, for example, these models scarcely consider whether people's representations have a *balanced* structure (Fig. 1). As a result, the transitive-chain model finds it difficult to directly handle representational changes caused by semantic factors, including the content effect, as does the probability heuristics model.

3.3. Summary

Mental models theory could be a promising account of internal mental representations used in syllogistic reasoning, but it suffers from a lack of specification, even in its latest form, leaving us desiring more detailed documentation and justification of the model's behavior. The probability heuristics model shows how effective probabilistic approaches are, even for logical tasks, but is not yet fully satisfactory, given that the nature of the heuristics is not explained in relation to mental representations. Although other theories such as set analysis theory and transitive-chain theory incorporate probability into their models, no theory has, until now, introduced the idea of probabilistic representations into syllogistic reasoning.

Although there have been several attempts to compare existing theories of syllogistic reasoning, including the mental model theory and the probability heuristic model, the method of evaluation varies considerably, and the results diverge: some studies found the probability heuristics model to be superior (Chater & Oaksford, 1999; Copeland, 2006), whereas others did not (Copeland & Radvansky, 2004; Khemlani & Johnson-Laird, 2012). Therefore, we need a comprehensive comparison of these models together with the probabilistic representation model.

4. Evaluation of the proposed model

The descriptive validity of the proposed model in predicting response patterns was assessed using actual experimental data. For the purpose of constructing a comprehensive theory, it is insufficient, albeit important, to examine whether or not a particular model predicts a particular type of syllogism, as I have done in the previous section. It is more important to evaluate the overall descriptive validity of models using an identical reasonable standard as inclusively as possible. Hence, I now evaluate the probabilistic representation model, in comparison with other signal models, in terms of how well it explains existing data as a whole from both quantitative and qualitative perspectives.[9]

Below, in Meta-Analysis 1, I compare the current model with other existing models that can predict the proportions of each conclusion type (i.e., A, I, E, and O) for each of 64 syllogisms. At present, only the probability heuristics model and transitive-chain theory provide such predictions. In this analysis, for fair comparison, I use the same data that the proponents of each model used to evaluate their own models. In Meta-Analysis 2, I conduct a more

[8] AA3 was later reclassified as a two-model syllogism, and then much later as a three-model syllogism, without any reason. This was pointed out and criticized by some researchers (e.g., Ford, 1994; Wetherrick, 1993).

[9] See Hattori (2016) for data used in Meta-Analyses 1, 2, and 3; and Experiments 1 and 2.

Table 3
Probabilities of constructing mental models as parameters of the p-mental model.

Syllogism	Successful (up to)			Unsuccessful
	1st	2nd	3rd	
One-model	P_{1M1}	–	–	$1 - P_{1M1}$
Two-model	P_{2M1}	P_{2M2}	–	$1 - (P_{2M1} + P_{2M2})$
Three-model	P_{3M1}	P_{3M2}	P_{3M3}	$1 - (P_{3M1} + P_{3M2} + P_{3M2})$

inclusive comparison using data from the literature: using the same set of extended datasets, all models were evaluated according to an identical standard. To make this comparison possible, the mental model theory is extended in line with the original idea to allow it to make quantitative predictions. After these comparisons, I examine how the model explains known qualitative phenomena in syllogistic reasoning, including immediate inference, Gricean interpretation, conversion, and figural effects. Thus, I reveal how the theory links syllogistic reasoning to other cognitive processes.

4.1. Meta-Analysis 1

The purpose of this analysis is to examine how well the current model performs on the same datasets used by previous models. The probabilistic representation model was compared with the probability heuristics model and the transitive-chain model. For the current model, predictions were derived by estimating the parameters that gave the best fit to the given data. These estimates were optimized by a quasi-Newton method using the "optim" function on R version 3.1.0 with "L-BFGS-B" options (Byrd, Lu, Nocedal, & Zhu, 1995). In this analysis, the goodness-of-fit was basically evaluated by Akaike's Information Criterion (AIC). A smaller score represents a better fit to the data. The AIC score was derived from the sum of the squared deviations of all (i.e., usually 64) syllogisms regarding a mean square deviation (MSD) of the choice rate for all five options (i.e., A, I, E, O, and N) as a respective square deviation of each syllogism. The root mean square deviation (RMSD) and Pearson's correlation coefficient (r) were also derived as indices of goodness-of-fit, as these are useful when two models have the same number of parameters. First, the predicted choice rate of each answer (including N, i.e., "No valid conclusion") for each syllogism was derived from a model. Next, MSD, RMSD, and r were calculated for each syllogism from the choice rate distribution.[10] Finally, the weighted average of the 64 (or fewer, depending on the experimental material) measure of each index was calculated. The weighting considered the number of participants (via Fisher transformation in the case of r).

4.1.1. Probability heuristics model

Chater and Oaksford (1999) evaluated their model using five experiments (a total of $N = 101$[11]). The current model was fitted to the same dataset, and the results were compared with their predictions (Chater & Oaksford, 1999, pp. 247–248). The fit of the current model and the probability heuristics model was equally good, with almost no differences: AIC = 17.7 vs. 17.9, RMSD = 0.829 vs. 0.824, respectively; and the correlation coefficient was almost the same: $rs = 0.973$ for both. Parameter values for the current model were as follows (those for the probability heuristics model are shown in the above paper): $x = 0.464$, $c = 0.895$, $u_A = 0.824$, $u_I = 0.492$, $u_E = 0.280$, $u_O = 0.285$.

4.1.2. Transitive-chain model

In the same way, the data from Experiment 1 ($N = 49$) of Guyote and Sternberg (1981) were used to compare the probabilistic representation model and the transitive-chain model. The former model predicts data with only five parameters (omitting one), as these data include only 45 out of 64 syllogisms, whereas the latter model has seven. The results show the superiority of the current model, considering the number of parameters: AIC = 13.8 vs. 16.6 (with parameter values $x = 0.499$, $c = 0.837$, $u_A = 0.872$, $u_I = 0.720$, $u_E = 0.131$), while the actual fit of the transitive-chain model was slightly better: RMSD = 0.077 vs. 0.046; $r = 0.979$ vs. 0.994, respectively.

4.2. An extension of the mental model theory: a parameterized model

Given that the mental model theory is currently the most comprehensive and representative theory of mental representations, it is important to compare its predictions with those made by the current theory. Unfortunately, however, this theory does not predict responses to syllogisms as a distribution of conclusion types, even in its latest form accounting for probabilistic reasoning (i.e., Khemlani et al., 2015), and it is thus impossible to compare the two models directly. Therefore, I constructed a parameterized model based on the idea of the original theory (the "*p*-mental model" for short).

In the mental model theory, the core machinery that generates variations in response patterns is the *search* for alternative models. Because each mental model searched is causative of particular response types, it is comparatively easy to construct a generative model for response variations. Several parameters that define the probabilities of the constructed models are listed in Table 3. For instance, in a two-model syllogism, the probability that people construct only one model (and fail to provide the correct answer) is P_{2M1}, and the probability that people construct the appropriate two models (and respond successfully) is P_{2M2}; the probability of other errors in the process of model construction is $1 - (P_{2M1} + P_{2M2})$.

I now illustrate how the *p*-mental model behaves using an example. In the case of AA2, which is defined as a two-model syllogism, the first mental model derives A, A′, I, and I′, according to the mental model theory (Johnson-Laird & Bara, 1984; Khemlani & Johnson-Laird, 2012).[12] Here, the prime symbol (e.g., A′) indicates the order in which end terms in the conclusion are converted (e.g., "All P are S") from the Aristotelian traditional order (e.g., "All S are P"). See Appendix A and Table A.1 for details on this topic.

If a participant only constructs the first mental model, but fails to search the second model, the model predicts that an A- or A′-statement will be output. I assume here that the *p*-mental

[10] The evaluation of the goodness-of-fit may be different from that in the meta-analysis of Chater and Oaksford (1999), who did not provide details of their method. The original probability heuristics model study does not provide proportions of N, and so these have been calculated by subtracting the proportions of A, I, E, and O from 100%.

[11] The virtual number of participants in Dickstein's (1978) Experiment 2 was treated as $N = 19$ here (actual $N = 76$), because each participant considered only one of four figures (a total of 16 out of 64 syllogisms) in this experiment.

[12] Confusingly, the notation of syllogistic reasoning differs considerably between papers following the traditional approach, including in the current article and papers by Johnson-Laird and his colleagues. For example, "conclusion I′ from syllogism AI1" corresponds to "conclusion Iac from syllogism AI2" in Table 7 of Khemlani and Johnson-Laird (2012).

Table 4
Results of Meta-Analysis 2 for data from eight experiments and of experiments 1 and 2.

ID	Study	Experiment	N	PRM			PHM			pMM		
				RMSD	*r*	AIC	RMSD	*r*	AIC	RMSD	*r*	AIC
D78-1	Dickstein (1978)	Expt 1	22	0.090	0.970	17.93	0.095	0.980	19.47	0.125	0.980	22.53
D78-2	Dickstein (1978)	Expt 2	19	0.093	0.961	18.18	0.090	0.968	18.40	0.121	0.926	21.51
JS78-2.1	Johnson-Laird and Steedman (1978)	Expt 2, 1st	20	0.103	0.978	19.97	0.106	0.971	20.72	0.119	0.968	21.25
JS78-2.2	Johnson-Laird and Steedman (1978)	Expt 2, 2nd	20	0.116	0.981	23.24	0.119	0.980	24.63	0.126	0.987	23.72
GS81-1	Guyote and Sternberg (1981)	Expt 1	49	0.077	0.979	13.78	0.122	0.952	18.69	0.116	0.969	19.44
JB84-3	Johnson-Laird and Bara (1984)	Expt 3	20	0.143	0.945	24.71	0.130	0.934	22.60	0.143	0.944	23.66
BBJ95	Bara, Bucciarelli, and Johnson-Laird (1995)	Adults	20	0.153	0.914	25.39	0.134	0.938	22.83	0.134	0.961	22.85
RNG01	Roberts et al. (2001)	–	56	0.106	0.952	20.02	0.084	0.971	17.66	0.106	0.930	19.35
	Weighted average	–	226	0.105	0.965	19.48	0.107	0.965	20.33	0.122	0.960	21.43
Exp-1	Experiment 1		88	0.090	0.959	8.08	0.133	0.942	8.17	0.102	0.976	14.15
Exp-2	Experiment 2		50	0.113	0.931	10.37	0.122	0.923	10.42	0.105	0.978	14.35

Note: PRM, PHM, and pMM indicate the probabilistic representation model, the probability heuristic model, and the new parameterized model based on mental model theory, respectively (see text). The underlined numbers indicate the best fit models. RMSD: root mean square deviation, *r*: Pearson's correlation coefficient.

model dismisses an I-type answer because, logically speaking, any A-statement implies the corresponding I-statement, and the former is always more informative than the latter. This is consistent with a common observation that very few people actually give an I-type answer when an A is available. In the same way, as the same relation exists between A′ and I′, E and O, and E′ and O′, I make the same supposition with regard to I′, O, and O′.

Otherwise, if a participant successfully constructs the second mental model, the *p*-mental model predicts that an N ("No valid conclusion") will be output, because the second model is consistent with E, E′, O, and O′, and it refutes A and A′, one of which has been derived at the previous stage of the first mental model. Therefore, A or A′ is derived with probability P_{2M1}, and N is derived with probability P_{2M2}. Other errors occur with probability $P_E = 1 - P_{2M1} - P_{2M2}$. For simplicity, the frequencies of other errors are assumed to be distributed uniformly across all incorrect responses. Consequently, when the options are A, I, E, O, and N, the predicted selection rates are P_{2M1}, $P_E/3$, $P_E/3$, $P_E/3$, and P_{2M2}, respectively.

The number of mental models and the conclusions derived by each are listed in Table 1. The former is based on Johnson-Laird and Bara (1984, Table 9–12), although there have been some criticisms on this topic (e.g., Ford, 1994; Wetherrick, 1993); the latter is based on Khemlani and Johnson-Laird (2012, Table 7).

4.3. Meta-Analysis 2

It may be considered unsatisfactory that Meta-Analysis 1 did not compare models simultaneously using the same datasets and the same single standard. Here, I compare three models under such conditions: the mental model theory as the most representative model based on internal representations, the probability heuristics model as the most influential probabilistic model, and the probabilistic representation theory as a promising integrated approach.

4.3.1. Method

The basic method of evaluation was the same as for Meta-Analysis 1, except for the following two points: all models were compared on the same extended datasets, and all models adopted the parameter estimation method described in Section 4.1. To obtain experimental data on syllogisms that provide the proportion of each response as a syllogistic conclusion (i.e., A, I, E, O, or N), I excluded data from experiments that used non-adult participants, that examined too few syllogism variations (less than half of the 64), and that did not have people derive a conclusion but instead had them examine the validity of each type of conclusion in a yes/no format (e.g., Rips, 1994). As a result, the set of eight experiments (with a total of $N = 226$) shown in Table 4 were chosen as the target of this analysis. This set is more inclusive than the data targeted in previous studies, including Chater and Oaksford (1999) and Khemlani and Johnson-Laird (2012).

Before the analysis, some datasets were slightly modified to adapt to the assumption of the models that the sum of all responses is 100%. In some experiments (JS78-2.1, JS78-2.2, JB84-3, BBJ95, and RNG01 in Table 4), in which participants did not make a choice from the given options but generated their own conclusion, the authors did not report the proportion of a few minor, unexpected responses. In such cases, data were complemented with N answers, as it is reasonable to assume that if a set of options were given to them, such participants would not have found any appropriate answer other than N.[13] Although participants in a generation task are not constrained by the order of terms in the conclusion (i.e., they can generate either an S–P or P–S conclusion), the difference was ignored in this analysis (following some predecessors, e.g., Chater & Oaksford, 1999) because some preliminary analyses indicated that the distinction has little effect on the results.

4.3.2. Results and discussion

First, to intuitively understand the goodness-of-fit of the current model, I drew graphs indicating the relation between the actual data and model predictions for each syllogism. The results are shown in Fig. 10. These data are the integrated set from the eight experiments taken from the literature. Although it is not ideal to simply total up all data, I thought this would be useful to grasp the overall fitness of the model, which is revealed to be quite good. Considering the difference in some aspects of the experimental method among studies including the population of participants, each model should be fitted to data from an individual experiment first, and then the results should be integrated by model to enable comparison. All other results described below are from such analyses.

Table 4 summarizes the results. The weighted average of AIC in Table 4 indicates that the probabilistic representation model produced the best performance (19.48), followed (albeit at a narrow margin) by the probability heuristics model (20.33) and the

[13] The effect of this modification on the target datasets is actually quite small, and the conclusions are not altered if they were treated as missing values: the average difference in the percentage of all response rates in terms of RMSD was 2.3, 1.4, 3.8, 1.2, and 2.1 for BBJ95, JB84-3, JS78-2.1, JS78-2.2, and RNG01 in Table 4, respectively.

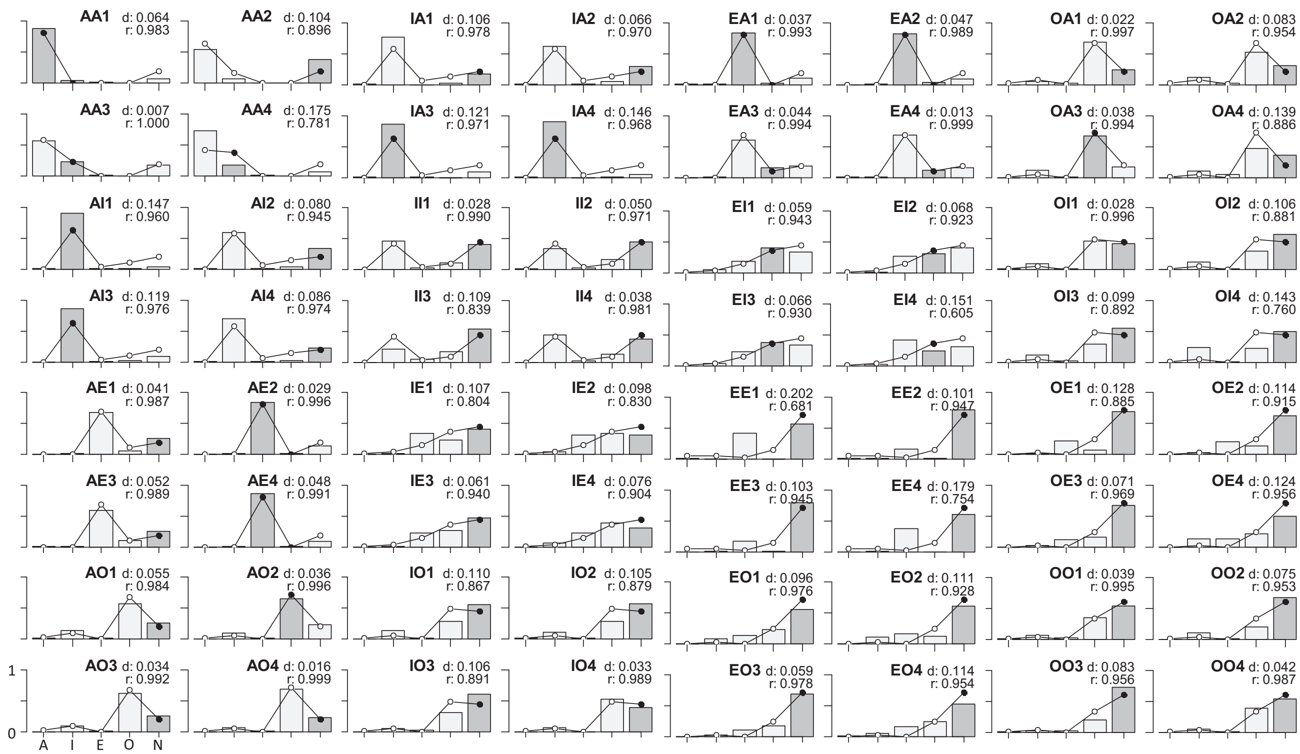


Fig. 10. Fit of the model to integrated data from eight experiments in the literature. Bars indicate data and lines indicate model predictions. Dark gray bars indicate valid conclusions. Numerical values in each figure indicate RMSD (d) and Pearson's correlation coefficient (r).

Table 5
Model parameters estimated for data from eight experiments in the literature and two new experiments (Meta-Analysis 2 and experiments 1 and 2).

ID	PRM						PHM						pMM					
	x	c	u_A	u_I	u_E	u_O	P_A	P_I	P_E	P_O	Pent	Perr	P_{1M1}	P_{2M1}	P_{2M2}	P_{3M1}	P_{3M2}	P_{3M3}
D78-1	0.478	0.905	0.897	0.525	0.189	0.210	0.806	0.398	0.178	0.152	0.050	0.006	0.930	0.392	0.578	0.298	0.295	0.401
D78-2	0.430	0.916	0.883	0.524	0.298	0.326	0.715	0.358	0.198	0.214	0.110	0.023	0.900	0.352	0.495	0.366	0.323	0.281
JS78-2.1	0.474	0.901	0.750	0.402	0.216	0.107	0.700	0.314	0.192	0.084	0.029	0.003	0.875	0.292	0.679	0.334	0.359	0.296
JS78-2.2	0.472	0.780	0.778	0.220	0.223	0.001	0.731	0.169	0.217	0.001	0.018	0.002	0.881	0.194	0.786	0.280	0.272	0.448
GS81-1	0.499	0.837	0.872	0.721	0.131	–	0.723	0.378	0.119	0.200	0.086	0.012	0.887	0.621	0.313	0.250	0.312	0.398
JB84-3	0.477	0.999	0.701	0.423	0.180	0.308	0.664	0.382	0.156	0.290	0.059	0.006	0.825	0.376	0.523	0.550	0.398	0.053
BBJ95	0.467	0.987	0.811	0.509	0.235	0.391	0.715	0.429	0.186	0.409	0.077	0.018	0.883	0.419	0.500	0.536	0.296	0.101
RNG01	0.465	0.979	0.841	0.643	0.408	0.601	0.733	0.519	0.307	0.485	0.083	0.026	0.831	0.494	0.329	0.565	0.237	0.163
All	0.473	0.914	0.828	0.548	0.248	0.342	0.726	0.394	0.202	0.266	0.070	0.014	0.873	0.391	0.514	0.419	0.302	0.253
Exp-1	0.499	0.805	0.832	–	–	–	0.673	–	–	–	0.114	0.019	0.832	0.560	0.274	0.386	0.324	0.176
Exp-2	0.485	0.779	0.788	0.542	–	–	0.560	0.377	–	–	0.183	0.021	0.560	0.377	0.200	0.200	0.183	0.021

Note: The top row indicates models and the ID indicates studies (see note to Table 4).

p-mental model (21.43). As all of the models compared here actually have the same number of parameters (i.e., six), the RMSD and r values can also be compared with one another. These alternative criteria converge to the same conclusion. Considered together, the fact that the probabilistic representation model scored best in five out of eight experiments in terms of AIC and was superior in terms of the other indices implies that it is the best overall model.

The parameter estimation results presented in Table 5 are also supportive of the proposed model. One of the merits of this model is that, unlike the other models, the meaning of parameters is clear in terms of cognitive processes. First, the coverage parameter c reflects how strongly people maintain the balancing principle in the task. The weighted average estimation was $c = 0.914$. This means that when people process A-type statements, it is highly probable that they implicitly assume that its conversion is also true. More precisely, a statement "All X are Y" leads to the assumption that "More than 90% of Y are X." This result is totally consistent with the set-size balancing hypothesis (Hattori & Nishida, 2009; Hattori & Oaksford, 2007).

Second, parameters u_A, u_I, u_E, and u_O reflect the max-heuristic. According to the analyses of Chater and Oaksford (1999), the max-heuristic predicts that the confidence in a conclusion should be in accord with the expected informativeness of the syllogisms with the different max premise types: $EI(\mathrm{A}) = 1.99$, $EI(\mathrm{I}) = 0.76$, $EI(\mathrm{E}) = 0.00$, and $EI(\mathrm{O}) = 0.05$, resulting in the order A > I > O > E. Again, most of the parameter estimation results in Table 5 agree with this prediction: $u_A > u_I > u_O > u_E$.

4.4. Immediate inference and Gricean interpretation

Although we have seen that the probabilistic representation model outperformed other signature models in quantitative evaluations, it is not yet clear whether the model's predictions are compatible with the findings revealed by previous studies on syllogistic reasoning. It is also worth analyzing, in some qualitative way, whether the model predicts several well-known characteristics of people's performance in syllogistic reasoning. I commence such analyses pragmatically, leaving the content effect to be considered in a later section of experiments (Section 5).

A view known as the *Gricean interpretation* of syllogistic quantifiers (e.g., Begg & Harris, 1982; Erickson, 1974; Newstead, 1995; Newstead & Griggs, 1983; Roberts, Newstead, & Griggs, 2001) partially provides the rationale for the model assumptions. The PPM of the probabilistic representation model for two propositions of a syllogism (Fig. 3) postulates the one-to-one correspondence between syllogistic statements and the Gergonne relations. Logically speaking, however, the correspondence is one-to-many, as mentioned in Section 2.2.1. In this sense, the model's correspondence assumptions about A-, I-, and O-statements violate the rules of mathematical logic. In conversation, "Some *X* are *Y*" implies "Some *X* are not *Y*," because we take it for granted that if the speaker knows "All *X* are *Y*," they would say so. Grice's (1975) *maxim of quantity*, which states that speakers should be as informative as possible, would reject D0, D1, and D2 in Fig. 3 as a representation of an I-statement ("Some *X* are *Y*"), and the representation is also shared by the corresponding O-statement ("Some *X* are not *Y*").

Evidence in support of the model's representation can be found in experiments using something called an *immediate inference* task. Newstead and Griggs (1983) tested people's immediate inference from a single sentence of syllogistic premises to investigate the interpretation of quantifiers. In their experiments, participants were given a statement like "All of the Ms. are Zs," and were asked to judge whether each of eight statements (e.g., "Some of the Zs are not Ms") followed logically from the given statement. Among four kinds of inferences they classified, the worst one was *subcontraries* (the other three were *contradictory*, *contrary*, and *subaltern* inferences): the majority of people falsely inferred that a true I-statement ("Some *X* are *Y*") or a true O-statement ("Some *X* are not *Y*") implies the truth of the other. This result is consistent with the probabilistic representation model. If we represent an I-statement by D3 in Fig. 3, then the corresponding O-statement is also true on the same representation (D3), and vice versa.

In the same experiments (Newstead & Griggs, 1983), about half of the participants (42% of TF questionnaire group in Experiment 1, and 57% in Experiment 2, respectively) judged a true A-statement ("All *X* are *Y*") to imply the truth of the converted O-statement ("Some *Y* are not *X*"). It is possible to interpret this result as suggesting that, according to a discrete view, half of the participants represented the A-statement as D1 and the other half represented it as D0. The result, however, is also compatible with another interpretation with a continuous view: most participants originally had a D1 representation as a prototype, but as the two representations in D1 almost overlap with each other ($c \approx 1$), about half of them are barely distinguishable from the case of D0.

More recently, Roberts et al. (2001) revealed that it is not Gricean interpretations alone that explain syllogistic reasoning data, but also *reversible Gricean interpretations*, which suggests a mixed strategy of accepting the converse (Chapman & Chapman, 1959) and Gricean interpretations. The reversible Gricean interpretation is similar to the PPM of the probabilistic representation model. The only difference is that the former postulates that A is D0, whereas the latter assumes that A is D1. Roberts et al. (2001) admit that simple inference tasks and syllogistic reasoning tasks need not be approached in exactly the same way (p. 176). Importantly, however, according to the probabilistic representation theory, there is no dissociation between simple inference tasks and syllogistic reasoning tasks. The model is compatible with all the data: Gricean interpretations, immediate inferences, and syllogistic reasoning, including the principle of accepting the converse that I now discuss.

4.5. Conversion as symmetry inference

As I noted earlier (Section 1), conversion is a form of symmetry inference, in which the current theory is especially interested. Illicit conversion (Chapman & Chapman, 1959; Sells, 1936; Wilkins, 1928) has received special attention from many theorists (e.g., Ceraso & Provitesa, 1971; Dickstein, 1978; Erickson, 1978; Newstead, 1989; Revlin & Leirer, 1978; Revlis, 1975a, 1975b; Roberts et al., 2001). Newstead (1989) concluded that "syllogistic reasoning errors are caused by misinterpretation of the premises" and "[c]onversion theory in particular seems to play an important part" (p. 91). In the latest extensive review of syllogistic theories, Khemlani and Johnson-Laird (2012) concluded that the illicit conversion theory correctly rejects participants' nonresponses better than any other theory. This means that this idea precisely captures a part, though not all, of people's behavior in syllogistic reasoning. Thus, I now examine how probabilistic representation theory is compatible with this phenomenon in two ways. One is empirical, and the other is theoretical.

First, I conducted a further analysis of Meta-Analysis 2 (Section 4.3) using the same data from eight experiments in the literature. Illicit conversion can be seen as emerging from the set-size balancing principle. If all the terms in a syllogism have the same probability, the relationship between any two terms becomes "symmetrical." As a result, conversion is no longer illicit. In the probabilistic representation model, the degree of balance is realized by the coverage parameter $c = 1$. In Meta-Analysis 2, the estimated values of this parameter suggest that the principle was largely maintained in the experiments. Here, apart from the best estimates, the robustness of this assumption is examined in view of how the variation in x and c affects the goodness-of-fit of the model. First, the model was fitted to the integrated data from the eight experiments, and the six parameters (i.e., x, c, u_A, u_I, u_E, and u_O) were estimated. Next, the model's goodness-of-fit was calculated for each of a 20×20 grid of parameters x and c while the other parameters were held constant. Fig. 11 illustrates the results. We can see that the most likely estimate for c is always close to 1, no matter what the value of x. This suggests that the evidence for the balancing principle is robust.

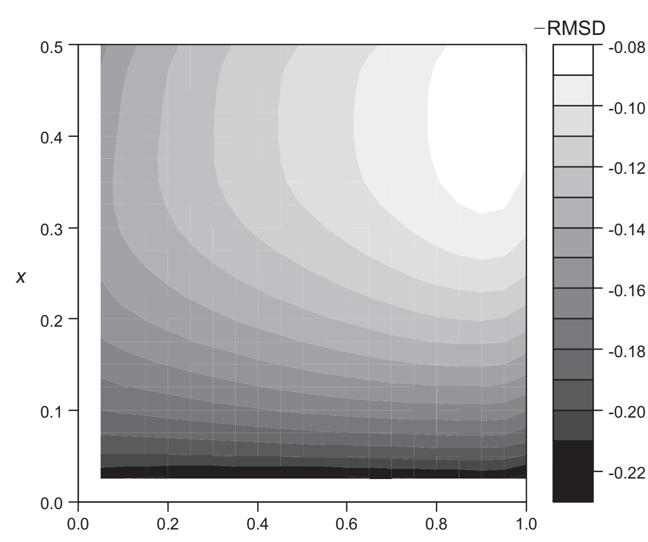


Fig. 11. The goodness-of-fit of the model as a function of parameters *c* and *x*. Other parameters were fixed as described in the text.

Second, I analyzed what parameters of the probabilistic representation model make it compatible with the conversion theory. According to Chapman and Chapman's (1959) principles, if there is at least one valid conclusion in the four syllogisms that have the same mood but different figures, participants tend to derive one of the valid conclusions. For the sake of simplicity, I assumed that if multiple conclusions were available by conversion, the most informative one would be adopted as a conclusion (i.e., in the order A > I > E > O), following the analyses of Chater and Oaksford (1999). For example, although the only logically correct answer of AA3 is I, conversion makes a more informative A conclusion available, and so A was assumed to be the answer predicted by conversion theory. Conversion theory predicts a particular answer for only 32 syllogisms, including four figures of AA, AI, IA, AE, EA, AO, OA, and EI. Comparing the model's predictions with those given by conversion theory, the rate of concordance among 32 syllogisms was plotted as a function of x and c (see Fig. 12). The results indicate that the conversion phenomenon is frequently observed (>85%) when $c > 0.9$ and $x > 0.25$ (approximately).

This result may not be of great surprise given that the two sets are identical when they maximally overlap under the constraint of the same set size. The important point is that this result enables us to make new predictions: when x and c are not in this area (i.e., the balancing principle is violated), conversion will no longer be observed. This will be examined in Section 5 (Experiments 1 and 2).

Finally, note that conversion makes the difference in the syllogistic figure (Fig. 2) ambiguous. As a result, conversion theory does not account for the *figural effect* discussed below, and this is one of the disadvantages of this theory, contrasting with the probabilistic representation theory.

4.6. Figural effect

Many theorists have mentioned that differences in the figure (see, Fig. 2) can affect the difficulty of syllogistic reasoning (e.g., Dickstein, 1978; Erickson, 1974; Espino, Santamaria, & Garcia-Madruga, 2000; Frase, 1966; Johnson-Laird & Steedman, 1978; Stupple & Ball, 2007). Generally, figure 1 is easiest and figure 4 is the most difficult. Frase (1966) reported this effect half a century ago. His experiments, which used 44 syllogistic yes/no questions consisting of 11 syllogistic moods (IA/O, IA/E, AI/E, AO/I, IE/I, II/I, EA/I, AE/I, EE/I, OE/O, and EI/O) and four figures, found that participants made the fewest errors on figure 1 syllogisms and the most errors on figure 4 syllogisms. Using 19 valid syllogisms (see Table 1), Erickson (1974) obtained a similar experimental result. Dickstein (1978) expressed caution about confounding factors including illicit conversion (Chapman & Chapman, 1959), and reached the same conclusion through careful analyses of screened syllogisms (i.e., EI and IE). A similar claim was made by Johnson-Laird and Steedman (1978), who ran experiments in which participants generated a conclusion for each syllogism. They pointed out that, when the two premises are B–A and C–B, participants prefer to generate a C–A type conclusion (that constitutes a figure 1 syllogism as a result, see Table A.1) to an A–C type conclusion (figure 4). Likewise, people prefer an A–C conclusion from A–B and B–C premises (constituting a figure 1′ syllogism in Table A.1, a logical equivalent of figure 1) to a C–A conclusion (figure 4′).

In evaluating how well a model predicts the figural effect, it is not obvious what measure should be used, despite the agreement on the conclusion that figure 1 is generally easier than figure 4. I examined how the model predicts the figural effect in two cases: using 19 valid syllogisms, following Erickson (1974), and using all 64 syllogisms. The difference in correct solution rate between figures 1 and 4 was examined as the two model parameters, x and c, varied. In the case of 19 valid syllogisms, the difference was always positive for any combination of parameters, indicating a stable figural effect whereby a figure 1 syllogism is always easier than a figure 4. The results from the complete set of 64 syllogisms are shown in Fig. 13. This figure shows that the figural effect is almost always observed, except for some special cases where c is approximately 0.5 and x is large. Given that c is almost always supposed to be high, as we have seen above, the model predictions are generally compatible with the existing data.

4.7. Working memory capacity

One of the distinctive features of the probabilistic representation model compared with other process models, including the mental models theory, is that it explicitly incorporates the assumption of working memory capacity. Working memory is considered to be an important component of cognitive processes (e.g., Baddeley, 2007), especially regarding conscious processing

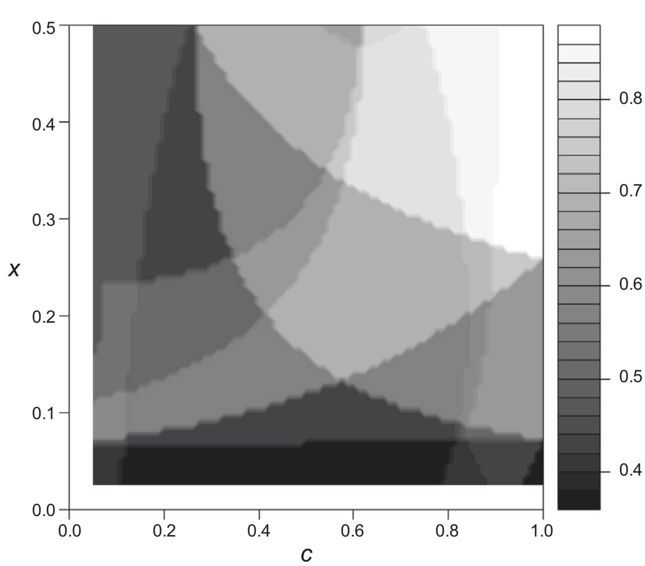


Fig. 12. The rate of agreement in predictions of conclusions as people's responses between theories of the probabilistic representation and illicit conversion.

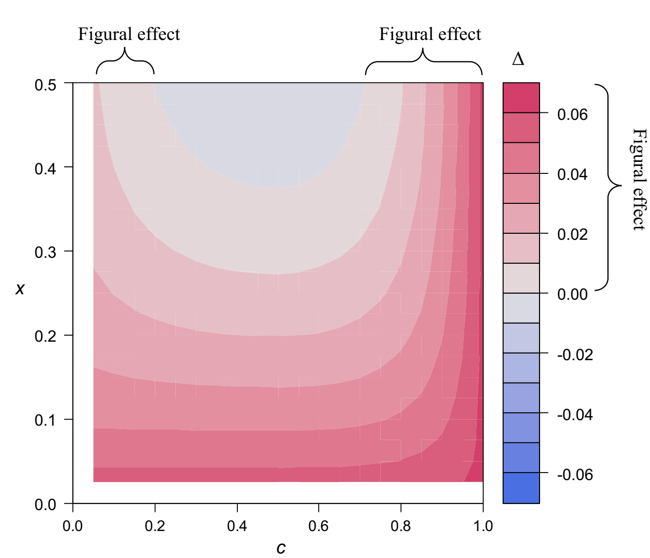


Fig. 13. The figural effect as the degree of superiority in average correct solution rate of figure 1 against figure 4 syllogisms for all 64 syllogisms.

including syllogistic reasoning, and working memory capacity must have some implications for reasoning processes. Therefore, it is important to evaluate the consistency of the model assumptions with the current findings on working memory.

Thus far, although I have hard-coded the sample size in SMM to be seven, this is not an intrinsic restriction of the model. I conducted a series of model fittings in which the sample size varied from 4 to 12 using the same dataset as in the conversion case in Section 4.5. As shown in Fig. 14, a sample size of six provided the best fit when evaluated based on RMSD, whereas it was seven based on Pearson's r. This result suggests that people sample six or seven instances in working memory to derive a conclusion to a syllogism. Because the size accords with Miller's (1956) magical number (i.e., seven plus or minus two), the result is supportive of probabilistic representation theory, which provides a model of syllogistic reasoning performed by people with a certain limitation of working memory capacity.

This evidence, however, may be controversial. Halford, Cowan, and Andrews (2007) claimed that the limit of working memory capacity, which is actually three to five chunks (wittily described as the "magical mystery four" by Cowan, 2010), reflects our capacity for attention in reasoning and restricts the relational representations that enable inferences to be made. Fig. 14 indicates that the fit of the model to the data greatly deteriorates when the number of samples is four, a pattern that does not seem to agree with recent findings on working memory capacity.

There are two ways to resolve this apparent dissociation. First, the capacity for processing can be dependent on the tasks to be performed. There may be a critical difference in cognitive load between recognizing a target as an instance of logical status (i.e., one SMM instance) and the process of a working memory task, in which people memorize each target precisely. There is also convincing evidence that the number of seven samples remains meaningful as our processing capacity when making decisions between many options (Iyengar & Lepper, 2000) or detecting a correlation between two events from covariation information (Kareev, 2000).

Second, the number of samples in SMM may not precisely correspond to the pure capacity of working memory. Cowan (2000, p. 112) pointed out that, while "the number seven estimates a commonly obtained, compound capacity limit," there is a possibility that "this number reflects a certain reasonable degree of chunking." The same thing might be true for SMM samples, as shown below.

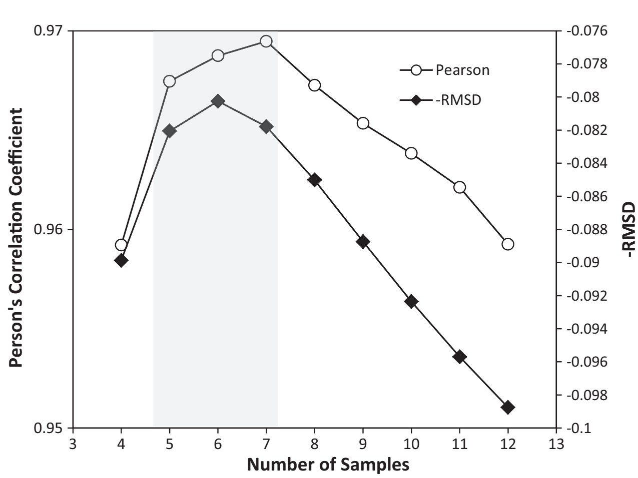


Fig. 14. The goodness-of-fit of the probabilistic representation model to data as a function of working memory capacity. Working memory capacity is assumed to be the number of samples in the SMM. Data for model fitting were taken from eight experiments in the literature, as introduced in Meta-Analysis 2.

The SMM sample distributes over the PPM, which basically consists of eight subsets corresponding to each logical status generated by the combination of three terms, S, M, and P. As this distribution is based on a random process, some samples can drop to the same PPM area; this actually happens in most cases. For example, in the case of AA1, the PPM of this syllogism does not have areas 2, 3, 4, and 6 (see Table 2 for the correspondence between the area number and the logical status), and only P_1, P_5, P_7, and P_8 can have a non-zero probability. If we use the best estimates obtained by Meta-Analysis 2 (i.e., $x = 0.473$ and $c = 0.914$), $P_1 = 0.43$, $P_5 = 0.04$, $P_7 = 0.05$, and $P_8 = 0.48$. The seven samples are thus dispersed throughout the four PPM subsets of AA1, but in reality, only two or three areas are generally occupied, especially when some probabilities are very small (i.e., P_5 and P_7 in this example). If we regard samples in the same area as being subject to chunking, seven samples do not in turn consume as much capacity as seven chunks. This was examined by the following simulation.

In the simulation, I examined the number of areas of a PPM occupied by seven SMM samples, which is assumed to be the capacity of working memory. The number of areas occupied by seven SMM samples was counted for each of 16 PPMs, with the model's parameters fixed to the best-fit estimates from Meta-Analysis 2. This sampling process was iterated 100,000 times, and the values were averaged by PPM. As a result, the total mean of the 16 PPMs (Fig. 6) was 3.10 (the minimum was 2.46 for AA2, AE13, and EA32, while the maximum was 4.84 for II). This analysis indicates that the model's behavior is also consistent with the findings of recent working memory studies.

4.8. Summary

In the literature, there are a few psychological theories of syllogistic reasoning that provide detailed quantitative predictions of people's performance in terms of a distribution of the proportion of answers. When such available models are compared with the probabilistic representation model in terms of their fit to the data, the latter model was always superior to the others, as shown in Meta-Analysis 1 (Section 4.1). Notably, the model also gave the best fit to the results reported by Guyote and Sternberg (1981) that have been almost ignored by previous meta-analysis studies, despite being the first quantitative model of syllogistic reasoning. When several signal models were compared simultaneously using a unified comprehensive dataset in Meta-Analysis 2, none was better than the probabilistic representation model (Section 4.3).

The probabilistic representation model does not only show a good fit to data. An obvious advantage of the model is that it is based on a theory of representation, and the model's parameters define reasoners' internal states in terms of representations. In this regard, it is distinctive among other probabilistic models. The model is an extension of Chapman and Chapman's (1959) conversion theory, but an elaboration allows the model to make novel predictions (Section 4.5), which are examined in the experiments described in the next section. The model also accounts for the effect of working memory capacity (Section 4.7).

The probabilistic representation theory is an integration of the probability heuristic model and the mental models theory, inheriting advantages from both theories. It also incorporates some accounts of syllogistic reasoning based on Gricean interpretation and p-validity. As a result, it not only quantitatively fits the data, but also qualitatively accounts for well-known phenomena in the literature, including immediate inferences (Section 4.4), illicit conversion (Section 4.5), and the figural effect (Section 4.6). Most importantly, it accounts for the background mechanisms of syllogistic reasoning. A psychological theory requires more than precise predictions. Without the internal mechanism, the theory is not able to make new predictions in other situations. Probabilistic representation theory is a theory at a representational level that handles mental mechanisms and accounts for data both quantitatively and qualitatively.

5. Experiments on the content effect

One of the merits of theories based on internal representation is that the effect of content is explained in terms of representational change. Probabilistic representation theory, in particular, expresses the change in internal representation by parameter values. Two experiments were conducted to examine the difference in the predictive powers of the models compared in the last section. Although the effect of content can be diverse, I concentrate here on an aspect of syllogistic reasoning concerning symmetry inference, an issue of particular interest for the current theory.

According to the set-size balancing hypothesis (Hattori & Nishida, 2009; Hattori & Oaksford, 2007), people assume that two target events are almost equal in their set sizes, unless there is effective information to override this belief. The coverage parameter c, which is responsible for the balancing assumption, only affects A-statements in syllogisms, and the degree of balance affects only syllogistic reasoning that includes A-statements. The difference in performance among four figures sharpens as the value of c decreases, because the degree of asymmetry between terms in an A-statement moves from complete symmetry ("All X are Y, and all Y are X" when $c = 1$) toward greater asymmetry ($P(Y|X) \to 0$ when $c \to 0$). Thus, probabilistic representation theory predicts that, if we somehow have people jettison the balancing principle, the syllogistic figure has a greater effect on the syllogistic reasoning performance. Such an influence of the figure is called an *inclusion effect* hereafter, and is a specific aspect of the figural effect, which is a somewhat ambiguous term (as mentioned in Section 4.6).

The predictions of the current model contrast impressively with the probability heuristics model, which does not predict any inclusion effect: no difference among figures for traditional syllogistic tasks with choice options. Although it is not necessarily easy to consciously inhibit the default response, it is known to be defeasible with the help of general knowledge stored in long-term memory, as shown by Hattori and Nishida (2009). Politzer (2011) proposed a similar idea of *natural syllogism* that falls in line with this. Applying this idea here, I examined whether the inclusion effect could be altered by a semantic manipulation whereby the relationship between two terms in an A-statement agrees with their logical relationship in terms of inclusion.

5.1. Experiment 1

5.1.1. Method

A total of 89 undergraduates from Ritsumeikan University (55 female and 34 male, age: $M = 20.2$, $SD = 1.0$) took part in the experiment to fulfill a part of their course requirements. Of these, one person did not follow the instructions correctly, and so their data were excluded.

In this experiment, only syllogisms that can have a greater inclusion effect were used as tasks in order to lighten the burden on participants and obtain clearer results. The potential of each type of syllogism (AA, AI, etc.) to exhibit the inclusion effect was defined by the average standard deviation of the proportion of each answer (A, I, E, O, and N) in all figures. Altering the value of c from 0.1 to 0.9 in steps of 0.1 (other parameter values were fixed at the best-fit estimates derived from Meta-Analysis 2), the index values were calculated and then averaged for each type of syllogism. As a result, five types of syllogism (AA, AI, IA, AE, and EA, in descending order) were chosen. Among these, 12 syllogisms (AA1, AA2, AA3, AA4, AI1, AI2, IA1, IA3, AE1, AE2, EA1, and EA3) that are different in the PPM were selected as tasks.

The terms used in each syllogism were carefully selected to nullify people's balancing assumption. Three levels of probability (low, middle, and high) were assumed. The "blood type Ph−" (low) was introduced as a fictitious category intended to remind people of the very rare actual blood type Rh−. The "pollen allergy" (mid) is actually quite common in Japan, but is no doubt less common than "having a cough" (high). This kind of general knowledge about prevalence was an attempt to override the default probabilistic representation and construct one that is consistent with the logical structure of a given task. For example, the AA1 syllogism was stated in a form with two premises, "All who have a pollen allergy (M-mid) have a cough (P-high)" and "All who have blood type Ph− (S-low) have a pollen allergy (M-mid)." Details of the material are shown in Appendix B. All the syllogistic tasks were printed in a booklet in a randomized order, and two versions (in which the order is reversed) were prepared and randomly assigned to the participants.

5.1.2. Results and discussion

The models were fitted to the data in the same way as in Meta-Analysis 2. As we can see from Table 6, the AIC again indicates that the probabilistic representation model (8.08) performed best, followed by the probability heuristics model (8.17) and the p-mental model (14.15). The RMSD and Pearson's correlation coefficient are also listed in Table 6. These are similar for the probabilistic representation model and probability heuristics model, because both can predict the data with only three parameters (as the data are from a part of all 64 syllogisms), whereas the p-mental model requires all six parameters. Generally, the results are consistent with those based on AIC.

The best-fit parameter estimates are given in Table 5. Note that the estimate of the coverage parameter, which is relevant to the balancing principle, is $c = 0.805$. The mean estimate of this parameter value in standard tasks from Meta-Analysis 2 was 0.912, which is significantly larger than in the current experiment, $t(7) = 3.72$, $p < 0.01$. This result indicates both the success of the experimental manipulation and the model's descriptive validity. I will discuss the degree of reduction in the value of c in Section 5.3.

5.2. Experiment 2

In this experiment, Experiment 1 was replicated with a wider variation of syllogisms.

5.2.1. Method

I used the same method as in Experiment 1, except for the following. Fifty undergraduate students from Kanazawa University (35 female and 15 male, age: $M = 20.8$, $SD = 1.0$) participated in this experiment, which was conducted as part of classwork in an elementary cognitive psychology class, albeit participation was on a voluntary basis. The tasks considered 32 out of 64 syllogisms: all syllogisms that can have the inclusion effect (i.e., syllogisms that include at least one A-statement in their premises) were selected, and II1, 2, 3, and 4 were also included. The tasks were conducted on a computer.

5.2.2. Results and discussion

Participants' responses to syllogisms IA2, II1, II2, and II4 were not properly recorded because of a bug in the computer program, and the results of these tasks were excluded from the analysis.

Once again, the models ranked the same in terms of the AIC (see Table 6): probabilistic representation model (10.374), probability heuristics model (10.422), and p-mental model (14.353). The estimated value of c was 0.779, i.e., the mean estimate of this

Table 6
Results of experiments 1 and 2.

Type	Experiment 1						Experiment 2					
	PRM		PHM		pMM		PRM		PHM		pMM	
	d	*r*	*d*	*r*	*d*	*r*	*d*	*r*	*d*	*r*	*d*	*r*
AA1	0.097	0.977	0.159	0.969	0.067	1.000	0.134	0.963	0.234	0.914	0.074	1.000
AA2	0.202	0.382	0.273	0.191	0.219	0.330	0.225	0.299	0.252	0.226	0.213	0.328
AA3	0.079	0.916	0.172	0.716	0.267	0.540	0.061	0.938	0.078	0.926	0.243	0.699
AA4	0.118	0.898	0.217	0.471	0.301	0.330	0.115	0.884	0.167	0.576	0.313	0.320
AI1	0.083	0.977	0.115	0.973	0.024	0.999	0.111	0.959	0.176	0.923	0.021	0.999
AI2	0.066	0.968	0.069	0.969	0.045	0.975	0.039	0.985	0.036	0.983	0.058	0.974
AI3	–	–	–	–	–	–	0.090	0.943	0.112	0.907	0.088	0.985
AI4	–	–	–	–	–	–	0.031	0.989	0.025	0.996	0.083	0.952
IA1	0.034	0.997	0.026	0.998	0.086	0.971	0.033	0.988	0.039	0.985	0.074	0.974
IA3	0.057	0.982	0.074	0.985	0.048	0.989	0.074	0.972	0.132	0.945	0.039	0.995
IA4	–	–	–	–	–	–	0.044	0.993	0.063	0.975	0.112	0.969
AE1	0.129	0.800	0.135	0.847	0.054	0.959	0.080	0.912	0.070	0.934	0.043	0.987
AE2	0.040	0.993	0.096	0.986	0.032	0.996	0.103	0.972	0.204	0.920	0.047	0.999
AE3	–	–	–	–	–	–	0.121	0.788	0.125	0.778	0.036	0.971
AE4	–	–	–	–	–	–	0.074	0.980	0.176	0.925	0.033	0.996
EA1	0.041	0.992	0.086	0.986	0.027	0.996	0.119	0.957	0.209	0.907	0.047	0.999
EA2	–	–	–	–	–	–	0.061	0.985	0.165	0.930	0.035	0.995
EA3	0.138	0.809	0.179	0.717	0.054	0.916	0.097	0.883	0.123	0.784	0.084	0.835
EA4	–	–	–	–	–	–	0.089	0.902	0.083	0.921	0.024	0.987
AO1	–	–	–	–	–	–	0.069	0.952	0.038	0.982	0.100	0.872
AO2	–	–	–	–	–	–	0.158	0.852	0.067	0.944	0.104	0.794
AO3	–	–	–	–	–	–	0.232	0.322	0.196	0.442	0.208	0.170
AO4	–	–	–	–	–	–	0.204	0.676	0.109	0.833	0.130	0.653
OA1	–	–	–	–	–	–	0.050	0.974	0.018	0.999	0.090	0.930
OA2	–	–	–	–	–	–	0.221	0.358	0.187	0.461	0.181	0.317
OA3	–	–	–	–	–	–	0.194	0.712	0.114	0.845	0.166	0.620
OA4	–	–	–	–	–	–	0.285	0.191	0.190	0.411	0.191	0.164
II3	–	–	–	–	–	–	0.034	0.993	0.040	0.987	0.100	0.856
All	0.090	0.959	0.133	0.942	0.102	0.976	0.113	0.931	0.122	0.923	0.105	0.978
AIC	8.080		8.170		14.146		10.374		10.422		14.353	

Note: The top row indicates models and the ID indicates studies (see, note of Table 4). Underlined numbers are the index values that indicate the best fit in each experiment. *d*: root mean square deviation (RMSD), *r*: Pearson's correlation coefficient.

parameter in standard tasks from Meta-Analysis 2 was again significantly larger than that from the current experiment, $t(7) = 4.622$, $p < 0.01$. As for Experiment 1, the results are supportive of the probabilistic representation theory.

5.3. Discussion

The results of these two experiments clearly show that people's representational change, which was manipulated by task materials in accordance with the set-size balancing hypothesis, alters their syllogistic reasoning. The variation in performance is predicted by the model as an adjustment of one parameter. This is the first demonstration of the inclusion effect, which is a clarified version of the figural effect.

One might, however, wonder at the degree of reductions in the value of c: estimated values did not reach as low as 0.5, but were around 0.8. The reason for this is not obvious, but there are several possibilities. First, it is possible that people's behavior is usually more conservative than expected. Similar results were actually obtained from previous studies that manipulated probability information in the Wason selection task. In experiments by Oaksford, Chater, and Grainger (1999), probability information was not sufficiently effective on people's performance. When Hattori (2002) estimated the probabilities of events that people conceive in the task from their performances, the results were conservative, although the trends pointed in the predicted direction. As these authors suggested, if a certain default heuristic is almost always useful in daily life, it may have become a hard-wired default option. Not being excessively sensitive to case-dependent information can be adaptive in the environment.

Second, it is also possible that some particular property of the stimulus words is responsible for manipulating people's probabilistic representations. In Hattori and Nishida's (2009) experiments, statements such as "A patient who is infected with X syndrome has an 80% chance of having a cough" drastically altered people's performance in probability judgment tasks. In this case, X syndrome and a cough are a disease and a symptom, and they have a clear causal relationship. While some statements used as stimuli in Experiments 1 and 2 can be regarded as causal rules, others such as "Some optimists have a pollen allergy" cannot. If probability information is more effective in the context of causality, the stimulus set might have made the results ambiguous.

Apart from these issues, the model itself might be too simple. Some assumptions, especially that $P(\mathrm{M})$ is the same in all syllogisms, might be too strong. The validity of each assumption should be carefully examined in future studies, and the model should be refined continuously.

6. Syllogisms with generalized quantifiers

Developments in the psychology of reasoning over recent decades (e.g., Evans & Over, 2004; Oaksford & Chater, 2007) have made the operational distinction between deduction and induction ambiguous (but, see, Rips, 2001). The traditional theoretical gap in philosophy between deductive and inductive reasoning has been bridged by several authors using probabilistic tools (e.g., Lassiter & Goodman, 2015; Oaksford & Chater, 2001; Tenenbaum, Kemp, Griffiths, & Goodman, 2011). In this regard, among several comprehensive psychological theories of syllogism, Chater and Oaksford's (1999) probability heuristics model has the distinctive advantage of dealing with generalized quantifiers (i.e., *most* and *few*), which are not regarded as being deductive from a logical point of view. Here, I extend the probabilistic representation model to deal with non-logical quantifiers, and compare it with the probability heuristics model.

6.1. Extension of the model

6.1.1. PPM

Although the quantifiers *most* (M) and *few* (F) are considered to express basically the same "logical" information as *some* (I), they have additional information. Therefore, I assume the Euler circle representing M and F to be D3 in Fig. 3, whereas the probability information for each of them includes an extra parameter (see Table 7). We can assume some kind of continuum from "No X are Y" (E) to "All X are Y" (A): E—F—I—M—A. Correspondingly, parameters m (i.e., *many*ness) and f (i.e., *few*ness) define the degree of overlap between two Euler circles for two terms in a statement. As shown in Table 7, it is assumed that m runs from 0 (I) to 1 (A) in the case of M, while f runs from 0 (I) to 1 (E) in the case of F. A set of PPMs for syllogisms with generalized quantifiers is the same as the traditional syllogisms shown in Fig. 6 (e.g., AI13 = AM13, etc.), although the probability assignment should be expanded for syllogisms including M or F, as given in Table 8.

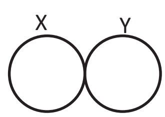
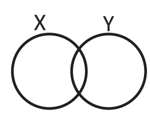
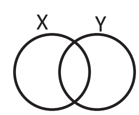
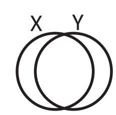
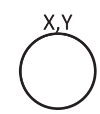
6.1.2. Generating a conclusion and its probability

The test order for the consistency of candidate conclusions is A, M, F, I, E, and O. This is based on the result that, under the rarity of events assumption, the order of informativeness is $I(\mathrm{A}) > I(\mathrm{M}) > I(\mathrm{F}) > I(\mathrm{I}) > I(\mathrm{E}) > I(\mathrm{O})$, as analyzed by Chater and Oaksford (1999). Additional criteria for testing the consistency of an SMM, and the theoretical probability that each conclusion is not inconsistent with the SMM, are as follows:

An **M**-conclusion is derived if there is at least one individual that is both S and P. The theoretical probability that the M-conclusion is consistent with an SMM is $(1 - P_1 - P_3)^n$.

An **F**-conclusion is derived if there is at least one individual that is not both S and P. The theoretical probability that the F-conclusion is consistent with an SMM is $1 - (P_1 + P_3)^n$. New semi-fixed parameters for the degree of confidence in a conclusion with M or F, as in the cases of A, I, E, and O, are also introduced.

Table 7
Generalized Quantifiers Expressed by Probability Parameters.

	No *X* are *Y*	Few *X* are *Y*	Some *X* are *Y*	Most *X* are *Y*	(All *X* are *Y*)
					
f:	$f = 1$	$0 < f < 1$	$f = 0$	–	–
m:	–	–	$m = 0$	$0 < m < 1$	$m = 1$
$P(X, Y)$:	$P(X, Y) = 0$	$P(X, Y) = \bar{f}x^2$	$P(X, Y) = x^2$	$P(X, Y) = xx^+$	$P(X, Y) = x$

Note. Probabilities of X and Y are expressed by a parameter x (i.e., $P(X) = P(Y) = x$), and parameters m and f indicate the degrees of *many*ness and *few*ness, respectively (see text in detail). Here, $\bar{m}$, $\bar{f}$, $\bar{x}$, and x^+ stand for $1 - m$, $1 - f$, $1 - x$, and $x + m\bar{x}$, respectively.

Table 8
Probabilities of all areas in each PPM with generalized quantifiers.

No	Name	Type	$P(\mathrm{S})$	$P(\mathrm{P})$	1	2	3	5
					$P(S, M, P)$	$P(S, M, \bar{P})$	$P(S, \bar{M}, P)$	$P(\bar{S}, M, P)$
5b	AM13	AM1,3	x	y	xx^+	0	$\bar{c}\bar{m}x\bar{x}$	$\bar{m}x\bar{x}$
5c	AF13	AF1,3	x	y	$\bar{f}x^2$	0	$\bar{c}x\bar{x}$	$x\bar{x}^+$
6b	AM24	AM2,4	x	cx	cxx^+	$\bar{c}xx^+$	0	$c\bar{m}x\bar{x}$
6c	AF24	AF2,4	x	cx	$c\bar{f}x^2$	$\bar{c}\bar{f}x^2$	0	$cx\bar{x}^+$
7b	MA12	MA1,2	cx	x	cxx^+	$c\bar{m}x\bar{x}$	0	$\bar{c}xx^+$
7c	FA12	FA1,2	cx	x	$c\bar{f}x^2$	$cx\bar{x}^+$	0	$c\bar{f}x^2$
8b	MA34	MA3,4	y	x	xx^+	$\bar{m}x\bar{x}$	$\bar{c}\bar{m}x\bar{x}$	0
8c	FA34	FA3,4	y	x	$\bar{f}x^2$	$x\bar{x}^+$	$\bar{c}x\bar{x}$	0
13b	ME	ME*	x	x	0	0	$\bar{m}x^2$	xx^+
13c	FE	FE*	x	x	0	0	$x^2\bar{x}^+/\bar{x}$	$\bar{f}x^2$
14b	EM	EM*	x	x	0	xx^+	$\bar{m}x^2$	0
14c	EF	EF*	x	x	0	$\bar{f}x^2$	$x^2\bar{x}^+/\bar{x}$	0
16b	MM	MM*	x	x	$x(x^+)^2$	$\bar{m}x\bar{x}x^+$	$\bar{m}^2x^2\bar{x}$	$\bar{m}x\bar{x}x^+$
16c	FF	FF*	x	x	$\bar{f}^2x^3$	$f\bar{f}x^2$	$x^2(\bar{x}^+)^2/\bar{x}$	$f\bar{f}x^2$
16d	MF	MF*	x	x	$\bar{f}x^2x^+$	$\bar{m}\bar{f}x^2\bar{x}$	$\bar{m}x^2(\bar{x}^+)$	$x\bar{x}^+x^+$
16e	FM	FM*	x	x	$\bar{f}x^2x^+$	$xx^+\bar{x}^+$	$\bar{m}x^2(\bar{x}^+)$	$\bar{m}\bar{f}x^2\bar{x}$
16f	MI	MI*; MO*	x	x	x^2x^+	$\bar{m}x^2\bar{x}$	$\bar{m}x^2\bar{x}$	$x\bar{x}x^+$
16g	IM	IM*; OM*	x	x	x^2x^+	$x\bar{x}x^+$	$\bar{m}x^2\bar{x}$	$\bar{m}x^2\bar{x}$
16h	FI	FI*; FO*	x	x	$\bar{f}x^3$	$x^2\bar{x}^+$	$x^2\bar{x}^+$	$\bar{f}x^2\bar{x}$
16i	IF	IF*;OF*	x	x	$\bar{f}x^3$	$\bar{f}x^2\bar{x}$	$x^2\bar{x}^+$	$x^2\bar{x}^+$

Note. Parameters m and f indicate the degrees of *many*ness and *few*ness, respectively (see text in detail). In this table, $\bar{m}$, $\bar{f}$, x^+, and $\bar{x}^+$, stand for $1 - m$, $1 - f$, $x + m\bar{x}$, and $\bar{x} + fx$, respectively. For $\bar{x}$ and y, see the note for Table 2.

6.2. Empirical tests: Meta-Analysis 3

The probabilistic representation model was compared with the probability heuristics model using data from Chater and Oaksford's (1999) Experiments 1 (AMFO) and 2 (MFIE) in the same way as in the previous evaluations. The fitness of the probabilistic representation model was inferior to that of the probability heuristics model in both experiments: RMSDs were 0.152 vs. 0.106 (AMFO) and 0.123 vs. 0.093 (MFIE); Pearson's r values were 0.899 vs. 0.965 (AMFO) and 0.767 vs. 0.859 (MFIE). AICs were 28.0 vs. 10.5 (AMFO) and 24.5 vs. 10.2 (MFIE), indicating much worse scores from the former, as it contains two more parameters than the latter.

There are some points to be noted regarding the tasks used in Chater and Oaksford's (1999) experiments, which are different from those used in traditional syllogistic experiments. In their experiments, half of the premises in the task contained probabilistic quantifiers (i.e., *most* or *few*), which clearly indicate that the target statement is not to be logically considered. In fact, fewer syllogisms have correct solutions from the logical point of view (i.e., 13 and 12 in the AMFO and MFIE tasks, respectively) compared with the traditional AIEO task (i.e., 19, see Appendix A). Such settings might have encouraged participants to think in a different way from ordinary syllogistic tasks. What is consistent with this view is the suggestion by Rips (2001) that people have qualitatively distinct ways of evaluating deduction and induction: a deductively correct argument is not an extreme form of an inductively strong argument.

While the distinction between deductive and inductive reasoning is still controversial (e.g., Feeney, 2007), it is no doubt an important issue for any current and future theories of syllogisms. Some anomalies observed even in the results of Chater and Oaksford (1999) might be relevant to this issue. In their Experiment 1 with AMFO quantifiers, many more participants (68.75%) selected the O-conclusion for syllogisms with OO-type premises than in experiments with standard deductive AIEO quantifiers (18.04%), a phenomenon that they called the global context effect. Participants' behavior changed in another way in their Experiment 2 with MFIE quantifiers: the selection trend was more ambiguous in this experiment than others. The percentage of the most selected conclusion types ranged from 25.0% to 75.0%, with an average of 39.53% (Table E1 in Chater & Oaksford, 1999). This rate seems to be significantly lower than for standard tasks, where the range is 12.03–92.41% with an average of 47.44% according to their Table C1. As a result, the fit of their model to the data is worse ($r = 0.65$) than the AMFO data ($r = 0.94$) and standard data ($r = 0.90$) according to their analyses. These phenomena seem to fall outside the scope of any current theories, and may have something to do with people's reasoning mode: deductive or inductive.

However, given that these results are from only two experiments with relatively few participants (i.e., 20 for each), and that the experiments were equipped with an insufficient set of syllogisms (i.e., only four out of six quantifiers, AMFIEO), the results of the current model evaluation are not definitive on this issue. In future, to construct a comprehensive model incorporating a possible distinction between deductive and inductive thinking modes and different types of processes, the method of the current model in deriving a conclusion including that based on a set of small samples in an SMM might require some elaboration.

7. General discussion

In this paper, I have proposed a model for syllogistic reasoning based on probabilistic representations. Constructing a mental representation based on individual elements that corresponds to a "probable" state of affairs, the model simulates a process to derive the most informative "logical" conclusion. The model exhibited a good fit to the available data, and its validity was confirmed. Table 9 summarizes all the results of empirical tests so far for the corresponding target models, picking up only AIC scores as a representative index of goodness-of-fit. This study can be characterized both as a true extension of the mental models theory that enables the quantitative prediction of data for syllogistic reasoning, and also as an attempt to elaborate the probability heuristics model at the algorithmic/representational level.

The results of parameter estimation for the coverage parameter c suggest a previously unknown connection among various types of reasoning and judgment. As c was estimated to be close to 1, S, M, and P are considered to be almost identical in size in our mind. In this relation, Hattori (2002) analyzed a version of information gain models and found that people assume the probabilities of the antecedent (p) and consequent (q) of a conditional "if p then q" in Wason's selection task to be almost equal. Similarly, Hattori and Nishida (2009) presented evidence that an error known as the base rate fallacy in probabilistic reasoning is caused by the balancing principle, and that this error disappears when the principle is blocked. Moreover, Hattori and Oaksford (2007) showed that a "fast and frugal" dual-factor heuristic embodying the principle is instrumental in covariation detection as a fundamental step in causal induction between two arbitrary events. All of these results indicate a particular aspect of general human cognition when people recognize the uncertainty of the real world. The balancing principle is a probabilistic interpretation of symmetry inference. Various biases and errors that have been identified in different areas of human thinking, including categorical and conditional deduction, induction, and probability judgment, appear to be unrelated to each other, but may be caused by a single common characteristic called symmetry.

Finally, I look at the justification for a particular assumption of the model. Random sampling, which is a mechanism for linking the probabilistic representation and derivation of a logical conclusion, can be questioned: is it simply an expedient mechanism for obtaining an output that fits the data, or is it assumed to be

Table 9
Summary of the all empirical tests: AIC as an index of goodness-of-fit of the model.

	PRM	PHM	pMM	TCM
Meta-Analysis 1: CO99-AIEO	17.9	17.7	–	–
Meta-Analysis 1: GS81	13.8	–	–	16.6
Meta-Analysis 2: Weighted average	19.5	20.3	21.4	–
Experiment 1	8.1	8.2	14.2	–
Experiment 2	10.4	10.4	14.4	–
Meta-Analysis 3: CO99-AMFO	28.0	10.5	–	–
Meta-Analysis 3: CO99-MFIE	18.6	10.2	–	–

Note: A smaller AIC score indicates better fitness. The top row indicates models and the ID indicates studies (see, note of Table 4). CO99-AIEO, CO99-AMFO, and CO99-MFIE indicate Chater and Oaksford's (1999) Meta-Analysis data using standard AIEO syllogisms, data from Experiments 1 (AMFO) and 2 (MFIE), respectively. GS81 indicates data from Guyote and Sternberg (1981).

psychologically real. The answer here is both yes and no. It is undeniable that at least part of our mental processes involves intrinsic fluctuations, and that a mechanism seen as a virtual random number generator is embodied in our minds (e.g., Glimcher, 2003). In this sense, the random sampling process in the model can be regarded as a model of a certain kind of psychological uncertainty. On the other hand, it may be unlikely that a particular person can be assumed to make a sample at "random," in the proper sense of the word, in the process of syllogistic reasoning. Rather, it would be more realistic to consider the random sampling function to be a sufficient model of reasoning performance of a certain group of people rather than a particular person. Although unpredictable uncertainty is surely involved in each individual's reasoning process, this may not be exactly the same as a purely random process. However, if we combine the behavior of a certain number of people, the global behavior is simulated by a random process with satisfactory accuracy.

8. Conclusions

The current study has presented a novel model based on probabilistic representations. The model was inspired by two preceding ideas proposed decades ago: probabilistic inference (Chapman & Chapman, 1959) and individual-based mental models (Johnson-Laird & Steedman, 1978). The proposed model was also elaborated using three concepts, some of which are relevant to previous theories of syllogistic reasoning: minimal constraints, small samples, and informativeness. The model exhibited a good fit to the data. As a result, the model is a real integration of the two most representative current theories, the mental model theory and the probability heuristic model (Chater & Oaksford, 1999). The most important implication of this study is the model's suggestion about (1) the *process* of syllogistic reasoning and (2) its relevance to other cognitive tasks or processes. The results suggest the procedural validity of probabilistic approaches: people first construct an intuitive probabilistic representation corresponding to the typical state of affairs described by the premises, and secondly, they construct an individual-based model with a small number of elements assuming minimal constraints on their logical relations. Thus, people try to convey information efficiently with such representations. The results also indicate that people observe the set-size balancing principle in syllogisms, as in other cognitive tasks, including hypothesis testing, causal induction, and probability judgment. This means that deductive reasoning is not a special cognitive process in the sense that it is exclusively modeled by logic, but is one of the ecologically justified processes regulated by adaptively rational probabilistic strategies.

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Appendix A. Validity of syllogisms and syllogistic figures

In this paper, I presuppose that at least one individual of *X* exists when asserting "All *X* are *Y*" (A) or "No *X* are *Y*" (E), following the traditional Aristotelian manner (i.e., the *existential presupposition*). If this presupposition is omitted, the number of syllogisms that have a valid conclusion is 15 instead of 19, as AA3, AA4, EA3, and EA4 do not have any valid conclusions. I adopt the Aristotelian criterion here because it is more natural from a psychological point of view, considering the fact that we would not usually say "All *X* are *Y*" when we already know there are no *X* at all. Johnson-Laird and Steedman (1978) regarded 27 syllogisms as having a valid conclusion (p. 67), and Chater and Oaksford (1999) regarded 31 syllogisms as being *probabilistically* valid (p. 203), because they ignored the order of terms in the conclusion. For example, AE1 does not have a valid conclusion according to the Aristotelian criterion, but if we are allowed to exchange S and P, "Some P are S" (I) would be a valid conclusion, while "Some S are P" (I), which has an allowed order, is not valid. Note, however, that this syllogism is no longer regarded as a Fig. 1 syllogism, but is instead a Fig. 4 syllogism (although the first and second premises are exchanged). That is, this syllogism is to be regarded as essentially EA4/O (Table A.1 shows XY1/Z′ = YX4/Z). I follow the traditional taxonomy of syllogisms to avoid confusion. Table 1 lists all 64 syllogisms and their logical conclusions.

According to traditional Aristotelian logic, syllogistic figures are defined as in Fig. A.1-I (including the conclusion, S and P). The predicate of the conclusion (P) is in the first premise (called a major term), and the subject of the conclusion (S) is in the second premise (called a minor term). In a series of experimental papers, Johnson-Laird classified syllogisms according to the location of terms in premises, as shown in Fig. A.1-III. In fact, this caused con-

Table A.1
Logical equivalence among various syllogistic figures.

Aristotelian	Exchanged	Converted	Johnson-Laird's	
XY1/Z	YX1′/Z	YX4/Z′	XY2^{JL}/Z′	(XbaYcb/Zca)
XY2/Z	YX2′/Z	YX2/Z′	XY3^{JL}/Z′	(XabYcb/Zca)
XY3/Z	YX3′/Z	YX3/Z′	XY4^{JL}/Z′	(XbaYbc/Zca)
XY4/Z	YX4′/Z	YX1/Z′	XY1^{JL}/Z′	(XabYbc/Zca)
YX1/Z	XY1′/Z	XY4/Z′	XY1^{JL}/Z	(XabYbc/Zac)
YX2/Z	XY2′/Z	XY2/Z′	XY3^{JL}/Z	(XabYcb/Zac)
YX3/Z	XY3′/Z	XY3/Z′	XY4^{JL}/Z	(XbaYbc/Zac)
YX4/Z	XY4′/Z	XY1/Z′	XY2^{JL}/Z	(XbaYcb/Zac)

Note. Symbols (e.g., XY1/Z) consist of the moods of the first premise (X) and second premise (Y), the figure (1), and the mood of the conclusion (Z). Figure numbers with a prime (e.g., 1′) indicate that the two premises are exchanged. Figure number with a superscript symbol JL indicates Johnson-Laird's original numbering system. Conclusion symbols with a prime (i.e., Z′) indicate that the order of terms is converted (i.e., P–S instead of S–P).

(I) Aristotelian Figures

(1)	(2)	(3)	(4)
M – P	P – M	M – P	P – M
S – M	S – M	M – S	M – S
S – P	S – P	S – P	S – P

(II) Exchanged Figures

(1′)	(2′)	(3′)	(4′)
S – M	S – M	M – S	M – S
M – P	P – M	M – P	P – M
S – P	S – P	S – P	S – P

(III) Johnson-Laird's Figures

(1)	(2)	(3)	(4)
A – B	B – A	A – B	B – A
B – C	C – B	C – B	B – C
A – C / C – A	A – C / C – A	A – C / C – A	A – C / C – A

Fig. A.1. Syllogistic figures defined in traditional Aristotelian logic (I), modified figures defined by exchanging the first and second premises from the Aristotelian figures (II), and Johnson-Laird's original figures (III). The former two fix the order of terms in the conclusion, but the latter allows conclusions in any order.

fusion because it had an independent numbering system under the same name of figures. This definition depends solely on the premises, and does not include the conclusion. As a consequence, correspondence between Aristotelian and Johnson-Laird's figures in regard to logical equivalence varies according to the order of conclusion terms. For example, now consider Johnson-Laird's Fig. 1 syllogism with A- and I-premises "All A are C" and "Some B are C" (AabIbc) in the form of Fig. A.1-III-(1). If it has an I-conclusion "Some C are A" (AabIbc/Ica), it is identical to AI4/I [Fig. A.1-I-(4)] (i.e., Fig. 4). However, if it has a conclusion "Some A are C" (AabIbc/Iac), it is actually logically equivalent to IA1/I (Fig. 1), as I now show. An AabIbc/Iac syllogism is identical to AI1′/I [Fig. A.1-II-(1′)]. Here, a figure number with a prime (e.g., 1′) indicates that it is logically equivalent to that figure, but the order of two premises has been exchanged, which means that AI1′/I is logically equivalent to IA1/I [Fig. A.1-I-(1)]. This kind of correspondence is listed in Table A.1.

Appendix B. Materials used in Experiments 1 and 2

Three levels of probability, low, middle, and high, were assigned to the syllogistic terms. Terms for low probability were "having blood type Ph− (B)" and "be exposed to phi-rays (R)." Terms for the middle probability were "having a pollen allergy (P)," "having X syndrome (X)," and "being an optimist (O)." Terms for the high probability were "having a cough (C)" and "having a stuffy nose (N)." The correspondence between syllogisms and the terms used is given in Table B.1.

Table B.1
Syllogistic terms used in experiments 1 and 2.

	S	M	P
AA1*	L (B)	M (P)	H (C)
AA2*	L (B)	M (P)	L (F)
AA3*	H (C)	M (P)	H (N)
AA4*	H (C)	M (P)	L (B)
AI1*	M (X)	M (P)	H (C)
AI2*	M (X)	M (P)	L (B)
AI3	M (X)	M (P)	H (C)
AI4	M (X)	M (P)	L (B)
IA1*	L (B)	M (P)	M (X)
IA2	L (B)	M (P)	M (X)
IA3*	H (C)	M (P)	M (X)
IA4	H (C)	M (P)	M (X)
AE1*	M (X)	M (P)	H (C)
AE2*	M (X)	M (P)	L (B)
AE3	M (X)	M (P)	H (C)
AE4	M (X)	M (P)	L (B)
EA1*	L (B)	M (P)	M (X)
EA2	L (B)	M (P)	M (X)
EA3*	H (C)	M (P)	M (X)
EA4	H (C)	M (P)	M (X)
AO1	M (X)	M (P)	H (C)
AO2	M (X)	M (P)	L (B)
AO3	M (X)	M (P)	H (C)
AO4	M (X)	M (P)	L (B)
OA1	L (B)	M (P)	M (X)
OA2	L (B)	M (P)	M (X)
OA3	H (C)	M (P)	M (X)
OA4	H (C)	M (P)	M (X)
II1	M (O)	M (P)	M (X)
II2	M (O)	M (P)	M (X)
II3	M (O)	M (P)	M (X)
II4	M (O)	M (P)	M (X)

Note. Stimuli with an asterisk were used in Experiment 1, and all stimuli were used in Experiment 2. L, M, and H indicate low, middle, and high probability, respectively. B, F, P, X, O, C, and N in parentheses indicate "blood type Ph−," "phi-ray exposure," "pollen allergy," "X syndrome," "optimist," "having a cough," and "having a stuffy nose," respectively. See details in text.

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