A new explanation of perceptual transparency connecting the X-junction contrast-polarity model with the luminance-based arithmetic model

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Abstract: The luminance-based arithmetic model of perceptual transparency was applied to explain Adelson-Anandan-Anderson’s X-junction contrast-polarity model, which classifies perceptual transparency on a phenomenological level. As a result, the reason why the apparent depth order of two surfaces is fixed in the configuration of unique transparency could be well understood. Moreover, the reason why the apparent depth order of two surfaces is reversible in the configuration of bistable transparency could also be well explained by introducing a new classification of perceptual transparency that differentiates full-layer transparency from object transparency. An experiment of bistable transparency revealed that object transparency is preferred to full-layer transparency. It also revealed two factors affecting perceptual transparency: average lightness or luminance of adjacent regions, and the difference in lightness or luminance between them. In addition, it appeared that Michelson contrast does not play a critical role in perceptual transparency.

Key words: perceptual transparency, arithmetic model, X-junction model, object transparency, full-layer transparency.

Perceptual transparency refers to the phenomenon that observers simultaneously see two surfaces at different depths in a single part of the given retinal image (Fuchs, 1923; Oyama & Nakahara, 1960).

Adelson-Anandan-Anderson’s X-junction contrast-polarity model
Perceptual transparency has been phenomenologically classified into two types: unique transparency and bistable transparency (Anderson, 1997). In the former, a transparent surface is always perceived in front of the other surface, while in the latter, the perceived depths of two surfaces are changeable and the surface in front appears to be transparent. These two types of perceptual transparency depend on the type of X-junctions. When contrast polarity along one edge is reversed over the X-junction while that along the other edge is preserved over the X-junction, unique transparency appears (Figure 1a). In contrast, bistable transparency appears when contrast polarity along both edges is preserved over the X-junction (Figure 1b). If contrast polarity along both edges is reversed over the X-junction, no transparency appears (Figure 1c).

According to Anderson (1997), this idea was first proposed by Adelson and Anandan (1990);
thus I would like to call it “Adelson-Anandan-Anderson’s X-junction contrast-polarity model.”

Metelli’s arithmetic model
Metelli (1974, 1985) proposed an arithmetic model to explain perceptual transparency.

Metelli’s figure shows unique transparency, in which a transparent disk always appears to be in front of a white-and-black background (Figure 2a). He attributed the transparent disk to an episcotister (i.e., a disk with open sectors) rotating above the critical flicker frequency.
(Figure 2b), assuming Talbot’s law of color fusion. When the reflectance is \(a, b, p\) and \(q\) for the white and black regions of the background and the lighter and darker halves of the transparent disk, respectively (Figure 2c), \(p\) and \(q\) can be given as follows:

\[
p = \alpha a + (1 - \alpha) t \quad \text{and} \quad q = \alpha b + (1 - \alpha) t,
\]

where \(\alpha\) is the angular proportion of the open sectors in the disk, and \(t\) shows the reflectance of the episotister. These equations can be solved and unknown \(\alpha\) and \(t\) are given by using \(a, b, p\) and \(q\):

\[
\alpha = (p - q)/(a - b) \quad \text{and} \quad t = (aq - bp)/(a - b - p + q).
\]

**The luminance-based arithmetic model**

Although this model has long been regarded as the most influential explanation of perceptual transparency, recent criticism raises fundamental difficulty (Beck & Ivry, 1988; Beck, Prazdny, & Ivry, 1984; Gerbino, Stultiens, Troost, & de Weert, 1990; Kasrai & Kingdom, 2001; Masin, 1997; Robilotto, Khang, & Zaidi, 2002; Singh & Anderson, 2002). The difficulty is that Metelli's \(t\) represents the reflectance of the virtual episotister, but not the reflectance or luminance of the real disk surface, which is suitable to describe perceptual transparency.

There are the following simple equations. They were first proposed by Gerbino et al. (1990) and developed by Gerbino (1994) as the “episotister luminance” model, and were later supported by Kasrai and Kingdom (2001). This luminance model is close to the “atmosphere” model proposed by Adelson (2000). When the luminance, not reflectance, is \(a, b, p\) and \(q\) for the white and black regions of the background and the lighter and darker halves of the transparent disk, respectively (Figure 2c), \(p\) and \(q\) can be given as follows:

\[
p = \alpha a + t \quad \text{and} \quad q = \alpha b + t,
\]

where \(\alpha\) represents the transmittance of the disk, while \(t\) indicates the luminance depending on reflection from the transparent surface. These relationships are represented diagrammatically in Figure 3a. These equations can be solved and unknown values \(\alpha\) and \(t\) are given by using \(a, b, p\) and \(q\):

\[
\alpha = (p - q)/(a - b) \quad \text{and} \quad t = (a - b + p + q)/(a - b - p + q).
\]
\[ t = (aq - bp)/(a - b). \]

**New rules given by the luminance-based arithmetic model**

The first equation implies that if \( a > b \), then \( p > q \) (Rule 1), because \( \alpha \) should be \( >0 \). When \( \alpha = 0 \), the disk is opaque. Moreover, if \( a > b \), then \( (a - b) \geq (p - q) \) (Rule 2), because \( \alpha \) should be \( \leq 1 \). If \( \alpha = 1 \) and \( t = 0 \), the disk cannot be seen. Furthermore, the second equation implies that if \( a > b \), then \( aq \geq bp \) (Rule 3), because \( t \) should be \( \geq 0 \).

**The luminance-based arithmetic model is consistent with Adelson-Anandan-Anderson’s X-junction contrast-polarity model**

Adelson-Anandan-Anderson’s X-junction model satisfies these rules. For the configuration of unique transparency (Figure 2c), if \( a > b \), \( p \) should be larger than \( q \), because contrast polarity along the vertical edge should be preserved over the X-junction, which satisfies Rule 1. Moreover, if \( a > b \), \( p \) should be larger than \( q \), because contrast polarity along the vertical edge should be reversed over the X-junction. The former inequality gives \( (a > p) \) and \( (q > b) \), because contrast polarity along the circumference edge should be reversed over the X-junction. These simultaneous inequalities give \( (a - b > p - q) \), which satisfies Rule 2. Furthermore, \( (a > b), (a > p), \) and \( (q > b) \) as just mentioned, multiplying the inequality \( (a > p) \) by \( q \) and the inequality \( (q > b) \) by \( p \), we obtain the relationships \( (aq > pq) \) and \( (pq > bp) \), respectively, because all variables are positive. These simultaneous inequalities give that \( (aq > bp) \), which satisfies Rule 3. Therefore, the configuration of unique transparency completely satisfies all these new rules.

For the configuration of no transparency (Figure 2e), if \( a > b \), \( p \) should be smaller than \( q \), because contrast polarity along the vertical edge should be reversed over the X-junction, which violates Rule 1. Secondly, if \( a > b \), \( (p - q) \) is negative and smaller than \( (a - b) \), which satisfies Rule 2. Thirdly, if \( a > b \), then \( q > p \). Thus we obtain the inequality \( aq > bp \), which satisfies Rule 3. Thus, the configuration of no transparency cannot satisfy all these new rules, although it satisfies two rules.

For the configuration of bistable transparency (Figure 2d), if \( a > b \), \( p \) should be larger than \( q \) because contrast polarity along the vertical edge should be preserved over the X-junction, which satisfies Rule 1. Secondly, if \( a > b \), \( (p - q) \) can either be smaller or larger than \( (a - b) \), which can either satisfy or violate Rule 2. Moreover, if \( a > b \), \( aq \) can either be larger or smaller than \( bp \), which can either satisfy or violate Rule 3. Therefore, the configuration of bistable transparency satisfies all these rules in some cases, but not in the others.

**Object transparency versus full-layer transparency**

However, it seems that the configuration of bistable transparency usually gives perceptual transparency. To overcome this discrepancy, the present study first proposes a new classification of perceptual transparency depending on whether transparency is given by a transparent object (object transparency) or a transparent layer (full-layer transparency) (Figure 4). The former shows that an object alone appears to be in

![Figure 4](image-url)
front of the background and to be transparent (Figure 4a) like Metelli’s figure, whereas the latter indicates that a layer including the object appears to cover the background and to be transparent all over the layer (Figure 4b,c).

The following are equations for an arithmetic explanation of Figure 3b or Figure 4b as an example of full-layer transparency:

\[ a = \alpha_1 c + t_1, \]
\[ b = \alpha_1 d + t_1, \]
\[ p = \alpha_2 c + t_2, \]
\[ q = \alpha_2 d + t_2, \]

where \( c \) and \( d \) show the luminances of the left and right parts of the background, respectively; \( \alpha_1 \) and \( \alpha_2 \) are the transmittances of the surround and the disk, respectively; and \( t_1 \) and \( t_2 \) are the luminances of reflection from the surround and the disk, respectively.

First, subtracting \( b \) from \( a \) gives \( \alpha_1(c - d) \), while subtracting \( q \) from \( p \) gives \( \alpha_2(c - d) \). Because \( \alpha_1 \) and \( \alpha_2 \) are positive, if \( a > b \), then \( p > q \). This result means that contrast polarity should preserve over the X-junction when examined along the vertical edge. Secondly, subtracting \( p \) from \( a \) gives \( (\alpha_1 - \alpha_2)c + (t_1 - t_2) \), while subtracting \( q \) from \( b \) gives \( (\alpha_1 - \alpha_2)d + (t_1 - t_2) \). This result cannot determine whether contrast polarity is preserved or reversed over the X-junction when examined along the circumference edge. Given that the layers are flipped where the surfaces of two rectangles appear to be in front (Figure 4c), contrast polarity is to be preserved, in turn, along the circumference edge. Thus, full-layer transparency can be generated, if contrast polarity along at least one edge is preserved over the X-junction. This constraint indicates that the configuration of bistable transparency can give the appearance of full-layer transparency in which two layers are reversible in depth.

Of course, the configuration of no transparency cannot give full-layer transparency, because contrast polarity along both edges is reversed over the X-junction, which violates the constraint of full-layer transparency. In contrast, unique transparency clears the constraint of full-layer transparency, but the visual system does not seem to select this appearance. This might possibly be because full-layer transparent scenes were not so frequently encountered in the process of the evolution of the human visual system, resulting in the preference for object transparency.

It is then suggested that the visual system first analyzes a given image to judge whether it is unique, bistable or no transparency, and then determines whether it is object transparency or full-layer transparency (Figure 5). When the image is judged to be unique transparency, the visual system adopts object transparency. In contrast, when the image is judged to be bistable transparency, either object transparency or full-layer transparency is selected, depending on unknown conditions.

**EXPERIMENT**

To examine what conditions determine the appearance of bistable transparency, the following experiment was conducted. In the author’s opinion, the luminance-based arithmetic model should adopt not luminance but perceived luminance (i.e., lightness), because perceptual transparency is a high-order psychological phenomenon. This experiment thus examined lightness as well as luminance.

**Method**

**Participants**

Ten naïve, paid participants, who had normal or corrected-to-normal vision, participated.

**Stimuli**

Forty different images of bistable transparency were used (Figure 6), in which a circle of 5.1 cm in diameter (consisting of two semi-circles) was drawn on the center of a square of 9.3 cm × 9.3 cm (consisting of two rectangles). Each image was printed using a high-resolution ink-jet printer in the center of an A4 piece of white cardboard. Each image had four regions, that is, two semi-circles (\( p \) and \( q \) in Figure 2d)
and two rectangles from which the half circles were removed (a and b in Figure 2d), where adjacent regions had different lightness. The lightness for painting was chosen from the following five, regularly selected Munsell values: N1.0 (almost black), N3.0 (dark gray), N5.0 (gray), N7.0 (light gray), and N9.0 (highly light gray near white), although color matching was conducted with the author’s vision using The Book of JIS Color Standards, Glossy Edition (JIS Z 8721) published by the Japan Color Research Institute in 1959. The luminances were 2.2 cd/m², 11.0 cd/m², 32.5 cd/m², 62.8 cd/m², and 92.0 cd/m², respectively. The luminance of the cardboard surface was 97.0 cd/m². To reduce the number of test stimuli, there were additional conditions that a is lighter than b and p is lighter than q, thus giving a darker appearance in the right half. These conditions gave 40 combinations, as shown in Figure 6, the parameters of which are listed in Table 1.

**Procedure**

Ten participants were individually tested. Each subject observed the 40 test stimuli once. They were asked to report the most predominant appearance among the following seven types of appearance: (A) the circle appeared to be in front of the two rectangles; (B) the surround of the circle appeared to be in front of the two rectangles;
rectangles and the circle appeared to be like an “open window;” (C) the left rectangle appeared to be in front of the square including the circle; (D) the right rectangle appeared to be in front of the square including the circle; (E) the square including the circle appeared to be in front of the two rectangles; (F) the two rectangles appeared to be in front of the square including circle; and (G) there was no perceptual transparency (Figure 7). A–D represented object transparency while E and F indicated full-layer transparency. There was no time restriction to respond. The order of presentations was randomized. The distance from the subject’s eyes to the stimuli was 40 cm. The room in which the experiment was conducted was lit using fluorescent lamps. The illuminance was 400 lx on the surface of the cardboard.
Results

Object transparency was most frequently reported (A: 16%; B: 14%; C: 16%; D: 37%; total: 82%) while full-layer transparency was much less frequently selected (E: 7%; F: 9%; total: 16%; Figure 8). Appearance G was rarely reported (2%). Moreover, there were large individual differences in reported appearances within each test stimulus. In addition, each subject reported all or most of the seven appearances.

How much the reported appearance accorded with the luminance-based arithmetic model was examined. The results show that more than half (52% for lightness; 64% for luminance) of the reported appearances violated some of the rules (Figure 9). Appearance D was reported most frequently (36%). This raises the possibility that darker (36%). This raises the possibility that darker

Table 1. Parameters of the 40 test stimuli. Numerical numbers show the Munsell values assigned to the four regions (a, b, p and q; e.g., “9” means N9.0)

| Test stimuli | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
|--------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| a            | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  | 9  |
| b            | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  | 7  |
| p            | 7 | 7 | 7 | 5 | 5 | 5 | 7 | 5 | 5 | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  | 3  |
| q            | 3 | 3 | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
pairs tend to be transparent and to be in front of the background, because the right half was in many cases darker than the left one (see Method). To examine this possibility, the mean lightness values or mean luminances of the reported pairs were compared with those of the unreported counterparts (if the surface \(a\) and \(b\) appeared to be transparent, \(a\) and \(b\) were the reported pair while \(p\) and \(q\) were the unreported counterpart). As a result, the reported ones were darker than the unreported ones (Figure 10). This difference was statistically significant (lightness:}

**Figure 8.** The result of the experiment, in which the proportion of reported appearances is shown for each test stimulus. D was reported most frequently (37%) while G was rarely reported (2%).

**Figure 9.** The proportion of the reported appearances in object transparency that violated some of the rules in the luminance-based arithmetic model. It was calculated with (a) lightness or with (b) luminance. In both cases, more than half of the reported appearances violated some of the rules.
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In addition, 69% of the reported pairs were darker than the unreported pairs. There is another possibility that the difference in lightness or luminance within each pair might be critical (Oyama & Nakahara, 1960). This possibility was examined and the result shows that the reported pairs had smaller differences in lightness or luminance than did the unreported counterparts (Figure 11). This difference was statistically significant (lightness: \( t(328) = 4.91, p < 0.01 \); luminance: \( t(328) = 8.12, p < 0.01 \)). In addition, for lightness values, 36% of the reported pairs had smaller differences than the corresponding unreported ones, while 15% of the former had larger differences than the latter. The other 49% of the former had the same difference as the latter. For luminance, the proportion was 68%, 32%, and 0%, respectively.

The Michelson contrast was also examined. For example, the contrast between two regions, \( a \) and \( b \), was defined as \( (a - b)/(a + b) \). As a result, surprisingly, the reported pairs had higher contrast than did the unreported counterparts (Figure 12). This difference was statistically significant (lightness: \( t(328) = 3.37, p < 0.01 \); luminance: \( t(328) = 2.91, p < 0.01 \)). In addition, for lightness, 34% of the reported pairs had lower contrast than the corresponding unreported ones, while 60% of the former had higher contrast than the latter. The other 5% of the former had the same contrast as the latter. For

\[ t(328) = 7.24, p < 0.01; \text{ luminance: } t(328) = 7.28, p < 0.01. \]  

\[ 3 \text{ The degree of freedom is 328 because the number of object transparencies reported was 329.} \]

Figure 10. Average (a) lightness and (b) luminance of the reported pairs and the unreported counterparts. For both units, the reported pairs were darker than the unreported ones. The error bar shows the SD.

Figure 11. The difference in (a) lightness and in (b) luminance within the reported pairs and within the unreported counterparts. The reported pairs had smaller differences in lightness or luminance than did the unreported ones. The error bar shows the SD.
luminance, the proportion was 39%, 61%, and 0%, respectively.

**Discussion**

The results were similar between lightness and luminance, so in the following discussion lightness and luminance will be treated as interchangeable.

The seven appearances (Figure 7), which were thought to be exhaustive in the condition of bistable transparency, were all reported by the subjects. However, object transparency was reported much more frequently than was full-layer transparency. As described above, this might possibly be because full-layer transparent objects were not so natural in the process of the evolution of the human visual system, resulting in the preference for object transparency.

It was found that more than half of the reported appearances violated some of the rules of the luminance-based arithmetic model. This indicates that the visual system does not always follow this model in bistable transparency, and that full-layer transparency is not necessarily selected even if object transparency gives invalid information. Instead, it was revealed that relatively dark regions tended to appear to be transparent and to be in front of the background (Tendency A). Moreover, when the difference in lightness or luminance between two adjacent regions was smaller, they tended to appear to be transparent (Tendency B).

Tendency B was already reported by Oyama and Nakahara (1960), who examined a cross shape of bistable transparency, as in Figure 1b. In contrast, when the Michelson contrast between two adjacent regions was higher, not lower, they tended to appear to be transparent (Tendency C). This finding is surprising because it has been proposed that regions of smaller contrast tend to be transparent (Tendency D: Anderson, 1997; Robilotto et al., 2002; Robilotto & Zaidi, 2004), although Robilotto et al. (2002) and Robilotto and Zaidi (2004) recorded “perceived” contrast, not the Michelson contrast.

For bistable transparency, changes in contrast are in many cases followed by changes in lightness or luminance. In reality, the present result can be consistent with Tendency D, because Tendency A inevitably accompanies higher contrast within dark regions than within light ones if the difference in lightness or luminance is constant. That is, it might be possible that Tendency A overwhelmed Tendency D, resulting in apparent Tendency C. Even if this is the case, Tendency D is weak.

It is therefore concluded that there are two main factors that cause particular adjacent regions to appear to be transparent and to be in front of the background. One factor is average lightness or luminance, where the darker regions tend to appear to be transparent. The other factor is the difference in lightness or luminance, where adjacent regions of the
smaller difference in lightness or luminance tend to appear to be transparent.

When subjects selected the darker pairs as the transparent surface, a clearly transparent object, like a neutral-density filter, was perceived. In contrast, when they reported the lighter pairs, a translucent object, like a thin piece of paper, was perceived. The latter appearance indicates that the visual system calculates not only the transmittance of the transparent surface \( \alpha \), but the lightness or luminance of its reflection \( t \), as mentioned by Kasrai and Kingdom (2001). In many cases, when the darker pairs appear to be transparent, Rule 3 is violated \( (t < 0) \), while it is not when the lighter pairs appear to be transparent or translucent. It can be thus speculated that the visual system implicitly calculates the lightness or luminance of reflection from the transparent surface, but explicitly applies it in the perceived image only when Rule 3 is kept \( (t \geq 0) \).

Sixteen per cent of the reported appearances were full-layer transparency. There was, however, no hint of what conditions are responsible for the selection of full-layer transparency. The novel finding was that full-layer transparency is not necessarily selected even if object transparency violates the rules of the luminance-based arithmetic model.

### Conclusion

The present study proposes a new model of perceptual transparency that the visual system first analyzes the given image to judge whether it is unique, bistable or no transparency, and then determines whether it is object transparency or full-layer transparency (Figures 4, 5). When the image is judged to be unique transparency, the visual system always adopts object transparency. In contrast, when the image is judged to be bistable transparency, either object transparency or full-layer transparency is selected. These ideas are consistent with the luminance-based arithmetic model.

The following experiment was conducted to learn the conditions in which the visual system selects object transparency or full-layer transparency when the image is bistable transparency.

The results showed that object transparency was preferred to full-layer transparency even in bistable transparency. It was also suggested that object transparency in the configuration of bistable transparency is affected by two factors: average lightness or luminance, and the difference in lightness or luminance.

### References


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