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# Apparent contraction of edge angles

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**Abstract.** The corner effect, the Münsterberg illusion, and the Café Wall illusion are explained by a model postulating that the corner effect is an orientation illusion specific to corner edges and that the perceived orientations of these edges are shifted toward angle contraction. It is also assumed that the effect is greatest when the corner edges show the same or similar edge contrast at the corner. This model yields three new types of illusions: the 'checkered illusion', the 'illusion of shifted gradations', and the 'illusion of striped cords'. Each of them gives many variations making a three-dimensional impression.

## 1 Introduction

Since the latter half of the 19th century, a variety of visual illusions have been discovered and extensively examined, but the figures of most of these are line drawings: eg the Müller-Lyer, Zöllner, Poggendorff, and Ponzo illusions. The figure of the Münsterberg illusion is an exception: it is a solid figure. Our visual environment usually consists of edges,[1] not of lines, and the study of visual illusions therefore requires more information about the visual characteristics of solid figures. In the present study I examine the known illusions of solid figures and demonstrate new types of illusion.

## 2 Explanations of the corner effect

The angles of a solid figure, such as a solid square, are underestimated and thus a right angle appears to be acute. Figure 1a shows an example. Every angle is a right angle, but each appears to be slightly acute. Figure 1b shows another example: a peri-stimulus time histogram (PSTH) used in electrophysiology illustrates this illusion, so that even a computer-drawn histogram made up of rectangles appears to be markedly distorted.

This phenomenon was first described by Pierce (1898) and examined by Moulden and Renshaw (1979), who named it the 'corner effect'. Pierce remarked that the corners of a white square 'bore into' the black area surrounding them. He attributed it to 'irradiation', a principle to account for the observation that "highly illuminated areas appear to be larger than they really are" (Helmholtz 1867/1962) (figure 2). How does irradiation produce the corner effect? Moulden and Renshaw (1979) gave an explanation: since the corner areas of a white square appear brighter than the other areas, their spreading effects are larger.

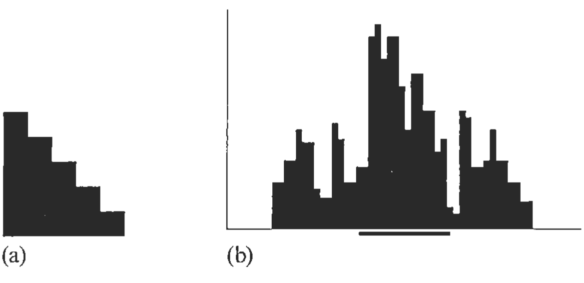


**Figure 1.** Examples of the corner effect: (a) 'illusion of a staircase'; (b) 'illusion of a peri-stimulus time histogram', in which the figure consists of rectangles of various sizes.

[1] An 'edge' indicates the border between two regions of different brightness.

**Figure 2.** An irradiation figure, in which the white square surrounded by a black background appears larger than the black square surrounded by a white background, though the former is the same size as the latter. According to Wade (1996), the history of irradiation goes back further than Helmholtz; it was described by Leonardo da Vinci.

This explanation is given for the white squares; but what about the black squares? They also show the corner effect, but Moulden and Renshaw did not refer to it. The present study provides an alternative explanation to the irradiation theory: the corner effect is an orientation illusion specific to corner edges, and the perceived orientations of the edges are shifted toward angle contraction. It is also proposed that the effect is greatest when the corner edges show the same or similar edge contrast at the corner. This simple hypothesis can easily explain the corner effect of black squares as well as white squares.

**3 Explanation of the Münsterberg illusion**

The present hypothesis—the apparent contraction of corner edge angles—explains the Münsterberg illusion as follows. As shown in figure 3, the black corners surrounded by white regions and the white corners surrounded by black regions located on the middle line are all underestimated, and thus each edge and its adjacent line are induced to tilt to the left. This produces the overall tilt according to the same principle as that observed in the twisted cords or the Fraser illusion (Fraser 1908).

**Figure 3.** A schematic explanation of how the Münsterberg illusion appears. Arrowheads indicate the sources of the illusion (corner effects).

This explanation is essentially the same as that given by Pierce (1898), who suggested that the white corners are solely responsible. In fact, white corners are not indispensable and black corners alone can yield this illusion (figure 4a). In addition to the corner effect of white corners, Moulden and Renshaw (1979) proposed another irradiation notion which they named the 'symmetrical effect': white regions spread into black squares and thus the middle line appears to tilt. The symmetrical effect, however, cannot explain the Münsterberg illusion with the reverse contrast (figure 4b). It is therefore concluded that irradiation is inadequate for explaining the Münsterberg illusion.

(a)

(b)

**Figure 4.** Evidence against irradiation explanations: (a) even if the white corners are replaced with arcs, the Münsterberg illusion remains; (b) the figure with the reverse contrast also gives the illusion.

The Münsterberg illusion disappears when the middle line is thick (Pierce 1898; Imai 1984). The thickness must not exceed about 10 min of arc (Gregory and Heard 1979). This characteristic is easily interpreted by the present model. The thick line changes the effective corners of edges (figure 5, arrowheads) and the corner effects of them just balance, ie the corner effects marked with a '+' give counterclockwise tilts

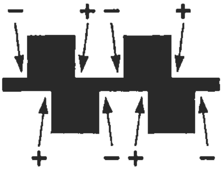


**Figure 5.** A schematic explanation why the Münsterberg illusion disappears in a figure with a thick line.

**Figure 6.** (a) The Münsterberg figure without the middle line; (b) the locations where the Morinaga misalignment illusion acts.

to the middle line, while those marked with a '−' produce clockwise tilts. The illusion thereby disappears.

If the middle line is removed (figure 6a), the strength of the Münsterberg illusion is very much reduced in comparison with the intact version (Moulden and Renshaw 1979; McCourt 1983; Imai 1984). McCourt (1983) attributed the reduction to the removal of the regions within which brightness induction can occur. This notion fits the evidence well (see the next section for details). The present study also gives a convincing explanation. When the middle line is removed, the aligned sides of facing black squares appear to be misaligned. This may be a variation of the Morinaga misalignment illusion.[2] Thus, the virtual lines drawn between the apices of black squares (white arrows in figure 6b) or between the apices of white squares (black arrows in figure 6b) appear to tilt in the reverse direction. Hence, the overall tilt is largely cancelled out.

Figure 7a shows an element or 'unit of direction' of the Münsterberg figure proposed by Fraser (1908), a notion which has been supported by several studies (Moulden and Renshaw 1979; McCourt 1983; Morgan and Moulden 1986). The present study supports

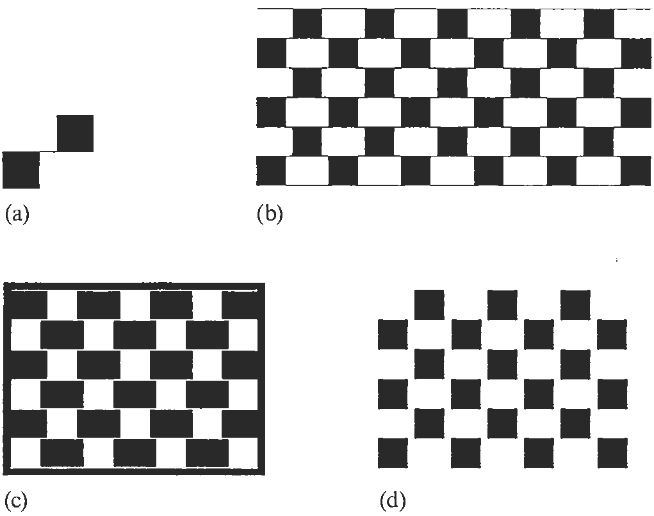


**Figure 7.** An element of the Münsterberg figure proposed by Fraser (1908) and the applied figure: (a) an element; (b) the 'illusion of a brick wall' made up of these elements; (c) figure with reverse contrast; (d) figure without real lines, showing the reverse illusion.

[2] When objects are placed diagonally, their aligned ends appear to be misaligned or to protrude a bit. This illusion was first reported by Morinaga (1941) and later detailed by Morinaga and Ikeda (1965) as a paradox in that "its direction is opposite to that intuitively expected from the Müller-Lyer illusion" (Day et al 1983).

this notion, and we propose a new illusory figure made up of these elements. Figure 7b shows the figure named the 'illusion of a brick wall', in which the white rectangles appear to swell. One may argue that this illusion favours the irradiation notion, ie white areas spread into black areas, but this is not the case. The figure with the reverse contrast also yields this illusion (figure 7c). Furthermore, when no real lines are drawn, we see a paradoxical illusion that black squares appear larger than the height of white areas (figure 7d). This illusion again may be a variant of the Morinaga misalignment illusion.

**4 Explanation of the Café Wall illusion**

Fraser (1908) found that the strength of the Münsterberg illusion increases when the middle line is replaced with a gray line. Gregory and Heard (1979) named it the 'Café Wall illusion' (figure 8), and argued that this illusion is the much more general case including the Münsterberg illusion. This illusion occurs when the luminance of the middle line (the mortar line in their terminology) lies between the luminances of dark and light squares, or when the middle line is not much darker than the dark squares or not much lighter than the light ones (Gregory and Heard 1979).

**Figure 8.** The Café Wall illusion, in which the middle line appears to slope down to the left.

They gave an explanation of this illusion using their notion of 'border-locking', which is described in the following terms: (i) regions of dissimilar luminance generate differential latencies in the retina; (ii) there are 'border-locking signals' which locate the border of edges; (iii) when differences in retinal delay are too great, the locking mechanism breaks down; (iv) in the Café Wall figure, regions of different luminance separated by a neutral gap are pulled together by the locking in compensation.

This theory contains too many hypotheses. There is no physiological evidence that regions of different luminance generate differential latencies in retinal cells or cortical cells. In addition, their account of the Münsterberg illusion is different from that of the Café Wall illusion; this is contradictory since the Münsterberg illusion belongs to the same class as the Café Wall illusion. These considerations therefore do not support the border-locking theory.

McCourt (1983) criticised the border-locking theory in a different manner and instead proposed the 'brightness induction' theory. Brightness induction is an illusion that the inducing fields of a vertical sine-wave or square-wave luminance grating (black and white) which sandwich a test field of space-average luminance (gray) induce the appearance of a second sine-wave or square-wave grating of equal spatial frequency, but of opposite phase (McCourt 1982). The basic idea accounting for the Café Wall illusion is that (i) the regions of the gray middle line sandwiched by white areas are induced to be black and those flanked by black areas to be white; (ii) the black squares and the line regions which are induced to be black form tilted line elements, as do their counterparts; (iii) a series of such tilts produces the overall tilt in the Café Wall illusion on the same principle as the Fraser illusion. This explanation fits the evidence well.

The bandpass filtering model proposed by Morgan and Moulden (1986) was also intended to explain the Café Wall illusion. This model shows that the bandpass filtering of the Café Wall figure yields a Fraser twisted cords illusion; ie the Café Wall illusion is attributed to the Fraser illusion. This model also fits the evidence well.

The present model also can explain the Café Wall illusion by adding the assumption that the magnitude of illusion at line elements produced by corner effects decreases according to the difference in luminance between the line and corners. For example, black corners (surrounded by white areas) render the greatest induction to a black line, but give weaker induction to a gray line. This model posits that the magnitude of illusion produced by black corners is reduced in the Café Wall figure as compared to the Münsterberg figure, but new induction sites are provided at white corners. Most critical is that the gray line extinguishes the negative effect of the Morinaga misalignment illusion existing between white corners. The Café Wall illusion thus has a greater effect than does the Münsterberg figure.

## 5 The 'checkered illusion'

Figure 9a demonstrates a new type of illusion, named the 'checkered illusion', in which the middle line appears to tilt to the left. It resembles the Münsterberg illusion but is different in that two black squares are paired and connected to each other at their corners. Corner effects are lost at the connection, since the effects of the two black corners cancel those of the two white corners. Thus, figure 9b shows that the former give clockwise tilts to the middle line (indicated by '−') while the latter render counterclockwise tilts (indicated by '+'). Thus, the remaining black corners located on the middle line alone exert illusory influences on the appearance, ie counterclockwise tilts ('+').

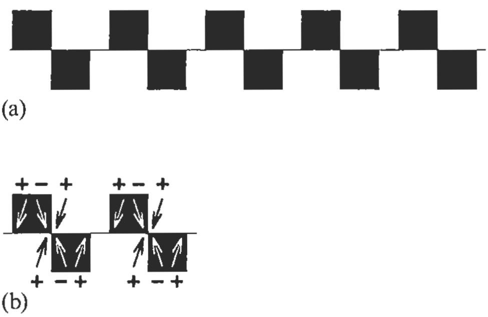



**Figure 9.** (a) The 'checkered illusion', deduced by the present model. (b) The corner effects involved in the 'checkered illusion', in which white arrows indicating the corner effects of black corners (giving clockwise tilts '−' to the middle line) and black arrows showing those of white corners (rendering counterclockwise tilts '+') are the same in number at the point where two black squares meet, thus canceling out each other at this point and leaving two white arrows, ie corner effects of black corners (giving counterclockwise tilts '+').

The 'checkered illusion' can be easily extended two-dimensionally (figure 10a), a feature specific to this illusion. A slight change in configuration makes a three-dimensional impression (figure 10b). Examples of elaborate '3-D' figures are illustrated in figure 11; they show a variety of visual effects. Needless to say, every figure consists of squares and every angle is a right angle.

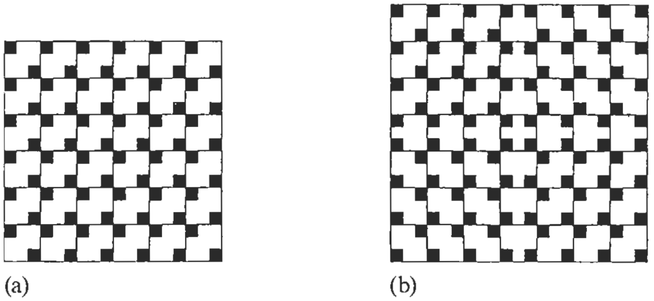
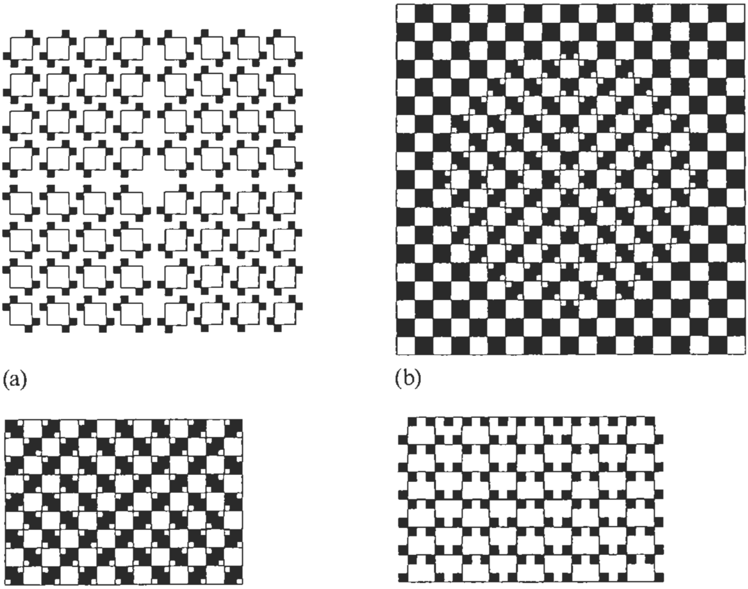



**Figure 10.** The 'checkered illusion' extended two-dimensionally: (a) horizontal lines appear to give counterclockwise tilts while vertical lines appear to render clockwise tilts; (b) a three-dimensional (bulging) impression made in a certain configuration. Both figures consist of squares and every angle is a right angle.

Right angles are not necessary for this kind of illusion. We can see the illusion with angles of 45° – 135° as illustrated in figure 12, in which each middle line appears to tilt to the left. At the remaining angles, the traditional orientation illusion, ie the apparent expansion of acute angles, disturbs the appearance of the 'checkered illusion'.



(a) (b) (c) (d)

**Figure 11.** Variations producing a three-dimensional impression: (a) the 'illusion of flying squares', in which each of the white squares appears to be distorted by attaching black squares and the special configuration of them makes a three-dimensional impression; (b) the 'illusion of a bulge'; (c) the 'illusion of a checkered flag', in which the flag appears to flutter in the wind; (d) the 'illusion of a castle wall', in which we perceive columns. Note that every figure consists of squares and contains no curves.

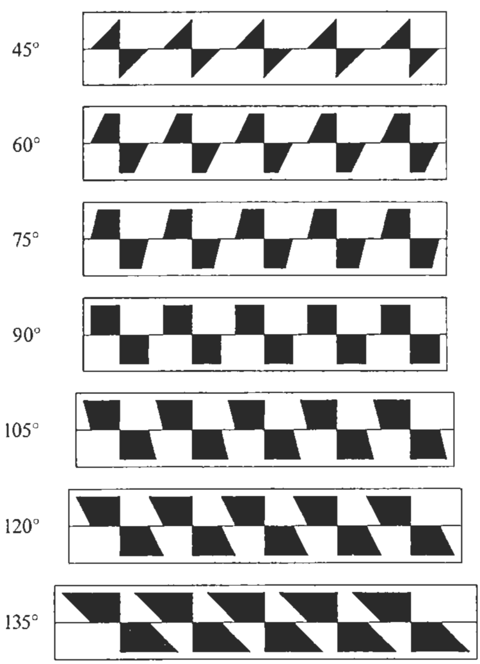



**Figure 12.** The 'checkered illusion' as a function of angle: 45°, 60°, 75°, 90°, 105°, 120°, and 135°, in each of which the middle line appears to tilt to the left.

To remove the effect of the orientation illusion, oblique components are drawn as short as possible (figure 13). This procedure has also evoked the 'checkered illusion' at the angles of 30° and 15°, ie each middle line appears to tilt to the left. The 'checkered

30°

15°

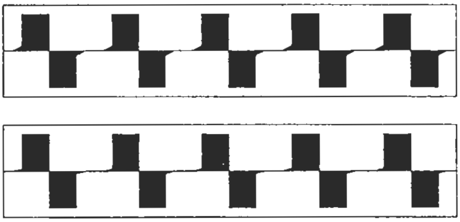


**Figure 13.** An illustration giving evidence for the illusion at 30° and at 15°, in which oblique components are drawn as short as possible; the middle line then appears to tilt to the left.

illusion' or the corner effect is therefore observed in a wide range of angles; further quantitative studies of this phenomenon are necessary.

The 'checkered illusion' was first noted and presented by Wade (1982) in his figure 3.2.33 (figure 14). He regarded it as a variation of the Münsterberg illusion. McCourt (1983) illustrated the checkered illusion in his figure 3, case 3 (figure 15). This figure renders a clockwise tilt to the middle lines, a reversal of the Café Wall illusion he was discussing.

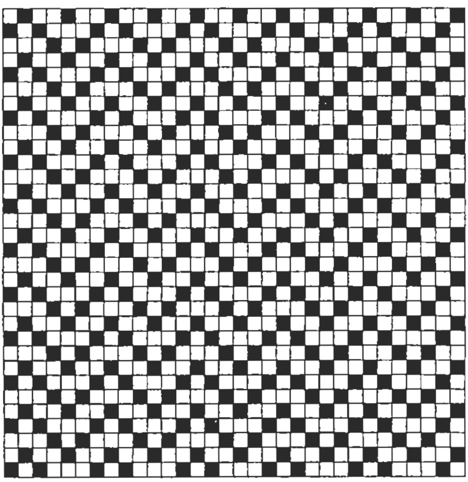


**Figure 14.** The 'checkered illusion' first given by Wade (1982) in his figure 3.2.33 (shown here with his permission).

**Figure 15.** The 'checkered illusion', in which the middle lines appear to tilt to the right [modified from McCourt (1983), figure 3, case 3].

## 6 The 'illusion of shifted gradations'

The present model deduces that shifted gradations of luminance (staircase luminance profiles) yield this type of orientation illusion at the border. As shown in each panel of figure 16, the uppermost border appears to tilt to the right, and this clockwise tilt alternates with the counterclockwise tilt. Figure 17 illustrates the explanation that each of the corners indicated by an arrow is formed by edges of the same contrast, yielding an underestimate of angles, whereas each of the other corners borders on different regions, giving a lesser corner effect. The overall border thereby appears to tilt to the left. Shifted triangular-wave (figure 18) or sine-wave gratings also show a similar illusion, of the clockwise tilt alternating with the counterclockwise tilt.

This type of illusion has a similarity to the graded version presented by Morgan and Moulden (1986) in their figure 6. This might be the first version of the 'illusion of shifted gradations'. The difference exists in the middle line: in their figure the middle line is made up of parallel black and white lines and the illusion strength seems much smaller.

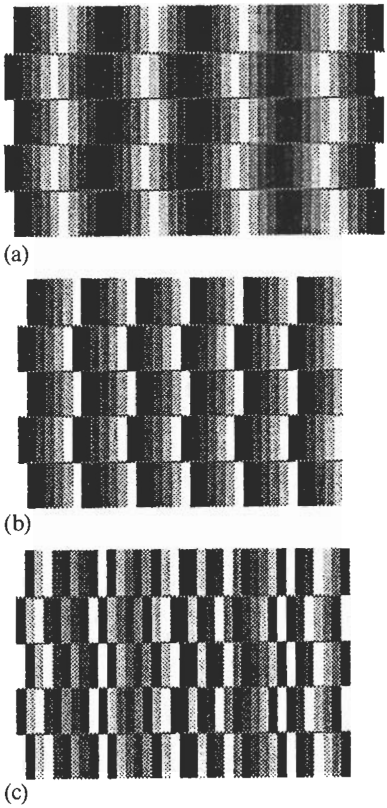


Figure 16. The 'illusion of shifted gradations' of six luminance steps: (a) triangular-wave changes in luminance; (b) repeated staircase changes in luminance; (c) random sequences in luminance. Each figure consists of rectangles but every case shows apparent tilts at the borders; the uppermost border appears to tilt to the right and clockwise tilt alternates with counterclockwise tilt.

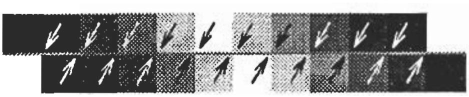


Figure 17. A schematic explanation of the 'illusion of shifted gradations': arrows indicate the corners formed by edges of the same contrast, which give the strongest corner effects; the border thus appears to tilt to the left.

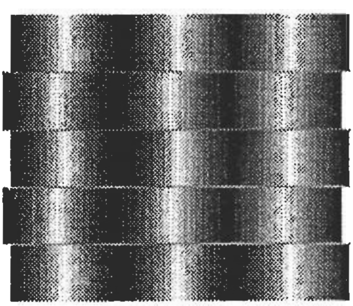


Figure 18. The 'illusion of shifted gradations' of continuous luminance changes (triangular-wave gratings), in which the uppermost border appears to tilt to the right and clockwise tilt alternates with counterclockwise tilt.

Like the checkered illusion, this illusion can also be extended two-dimensionally. Figure 19 shows that the physically straight borders appear to be curved so that the black squares appear to be smallest and the white squares located at the white crosses appear to be largest.

## 7 The 'illusion of striped cords'

Figure 20a shows another version of this type of orientation illusion. When light-gray and dark-gray squares are arranged like a checkerboard, the middle line made up of a series of alternating white and black lines appears to tilt to the left. This illusion can be explained with the assumption described before that the strength of illusion at line

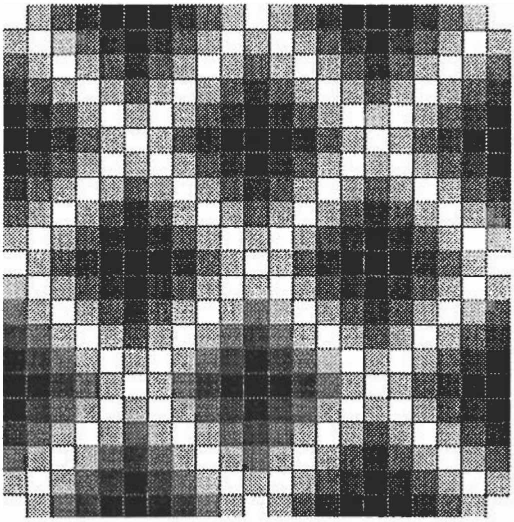
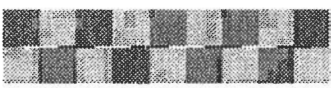
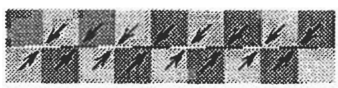


**Figure 19.** The 'illusion of shifted gradations' extended two-dimensionally, in which the straight borders appear to be curved so that black squares appear to be smallest and the white squares at the white crosses appear to be largest.

elements produced by corner effects decreases with decreasing difference in luminance between the line and corners. Figure 20b shows the explanation that each of the corners indicated by an arrow gives a stronger corner effect than each of the other corners, ie the light-gray corners and dark-gray ones give stronger corner effects to white lines and black lines, respectively. The overall middle line thereby appears to tilt to the left. This illusion can also be extended two-dimensionally (figures 20c and 20d).



(a)



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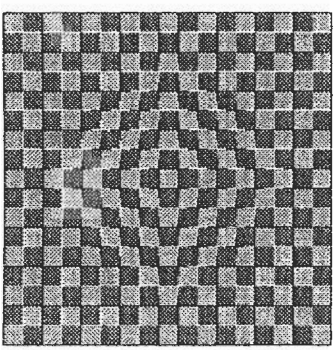

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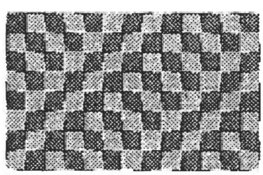

(d)

**Figure 20.** The 'illusion of striped cords'. (a) The figure consists of light-gray and dark-gray squares which are arranged like a checkerboard and of a middle line made up of a series of alternating white and black lines; the middle line appears to tilt to the left. (b) A schematic explanation of this illusion: each of the corners indicated by arrows gives a stronger corner effect than each of the other corners. (c) and (d) Examples of the two-dimensional extension of this illusion: the 'illusion of a bulge' as in figure 11b and the 'illusion of a checkered flag' as in figure 11c, respectively.

## 8 General discussion

This study has drawn attention to illusions in solid figures such as the Münsterberg figure, and proposed a simple model: the corner effect is an orientation illusion specific to corner edges, and the perceived orientations of the edges are shifted toward angle contraction. The model has explained the corner effect, the Münsterberg illusion, and the

Café Wall illusion. Furthermore, it has allowed the development of three new types of illusion, the 'checkered illusion', the 'illusion of shifted gradations', and the 'illusion of striped cords'.

The corner effect is thus thought to be one of a class of orientation illusions, but quite different from the traditional orientation ones such as the Zöllner illusion. First, the former appears at a wide range of angles including a right angle (see figures 12 and 13) whereas the latter occurs at angles of $10°-60°$ or $120°-170°$ (Morinaga 1933; Wallace and Crampin 1969). Second, the angle giving the largest illusion seems to be unclear for the former illusion, whereas it is $10°-30°$ or $150°-170°$ for the latter. Third, angles are underestimated at any angle for the former, while overestimated at acute angles and underestimated at obtuse angles for the latter. Finally, the corner effect can be seen only at the corners of solid figures, whereas the Zöllner illusion is observed even when interacting objects are not connected to each other (Wallace 1969; Oyama 1975; Weale 1978).

Irradiation has been found to be inadequate to explain the Münsterberg illusion, since it appears even when white corners are removed or the contrast of the figure is reversed. However, we can see the effect of irradiation in several cases. For example, it seems that the Münsterberg figure with a black middle line (figure 3) shows a larger illusion than that with a white one (figure 4b). In addition, it also seems that the strength of the 'illusion of brick wall' is greater in the figure with black squares (figure 7b) than in the figure with white squares (figure 7c). Irradiation thus affects the perception of some of the illusory figures, but not critically.

The brightness induction theory proposed by McCourt (1983) and the bandpass filtering model proposed by Morgan and Moulden (1986) explained the Münsterberg and Café Wall illusions. They can also account for the 'checkered illusion' in the same manner as the Münsterberg illusion and explain the 'illusion of shifted gradations' in the same manner as the Café Wall illusion. Furthermore, they may probably account for the 'illusion of striped cords'. However, they cannot explain the corner effect.

In conclusion, the present model gives a satisfactory explanation of a variety of orientation illusions specific to solid figures, and the mechanism underlying the corner effect is of great interest for future work.

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